

Supplementary Information for Random graphs and real networks with weak geometric coupling

Jasper van der Kolk^{1,2}, M. Ángeles Serrano^{1,2,3}, Marián Boguñá^{1,2}

¹Departament de Física de la Matèria Condensada, Universitat de Barcelona,
Martí i Franquès 1, 08028 Barcelona, Spain

²Universitat de Barcelona Institute of Complex Systems (UBICS),
Universitat de Barcelona, Barcelona, Spain

³ICREA, Pg. Lluís Companys 23, E-08010 Barcelona, Spain

Contents

S1 Supplementary Figures

1

S1 Supplementary Figures

In the following section we will give the embedding and renormalization results for several real networks. The network properties can be found in Tab. 1 in the main text. Similarly, a brief description of the network can be found in App. B.

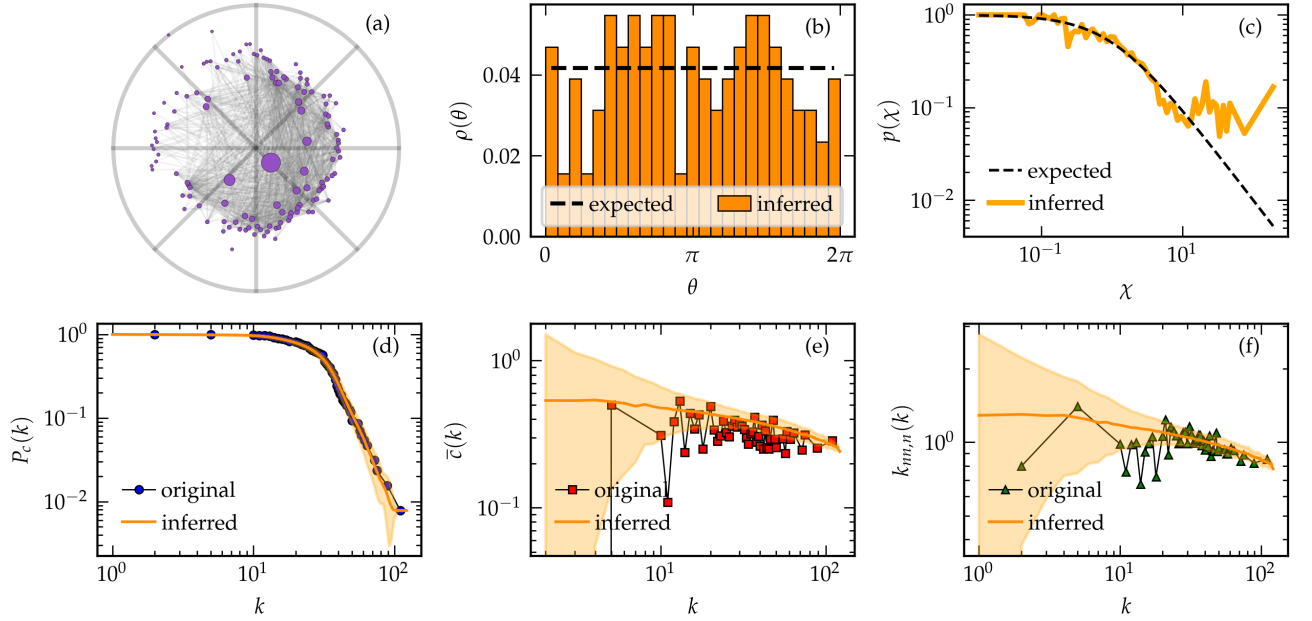


Figure S1: Summary of the results of Mercator for the Foodweb-Eocene network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 100% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

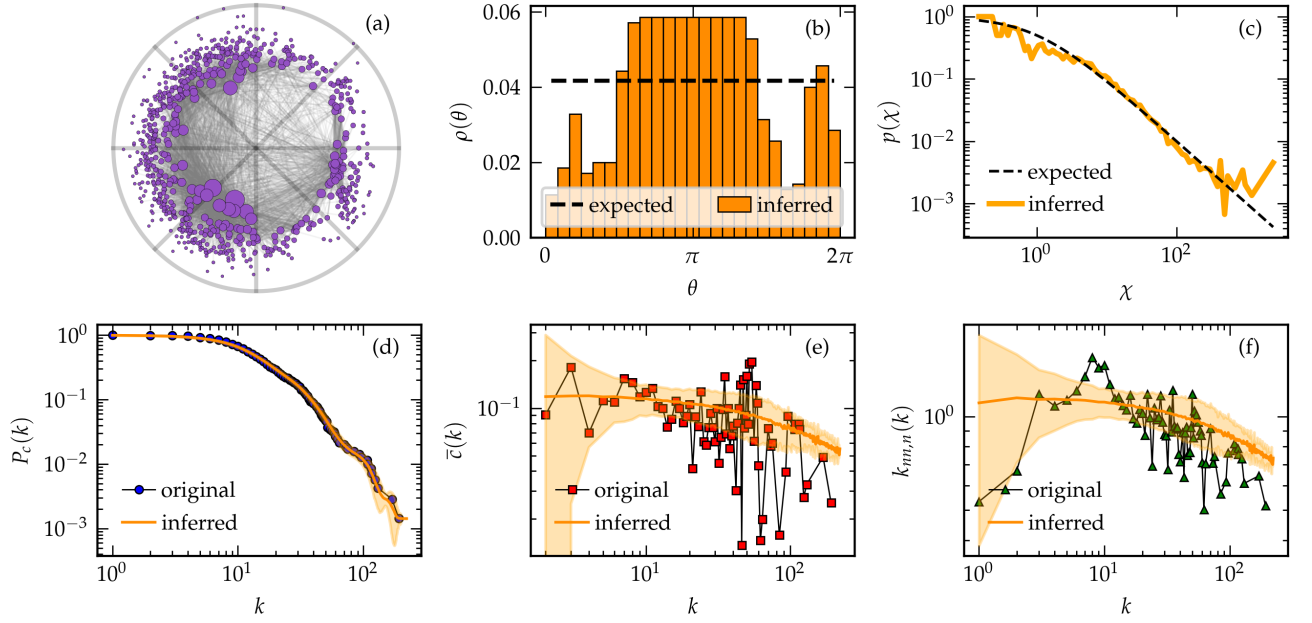


Figure S2: Summary of the results of Mercator for the Foodweb-Wetland network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 50% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

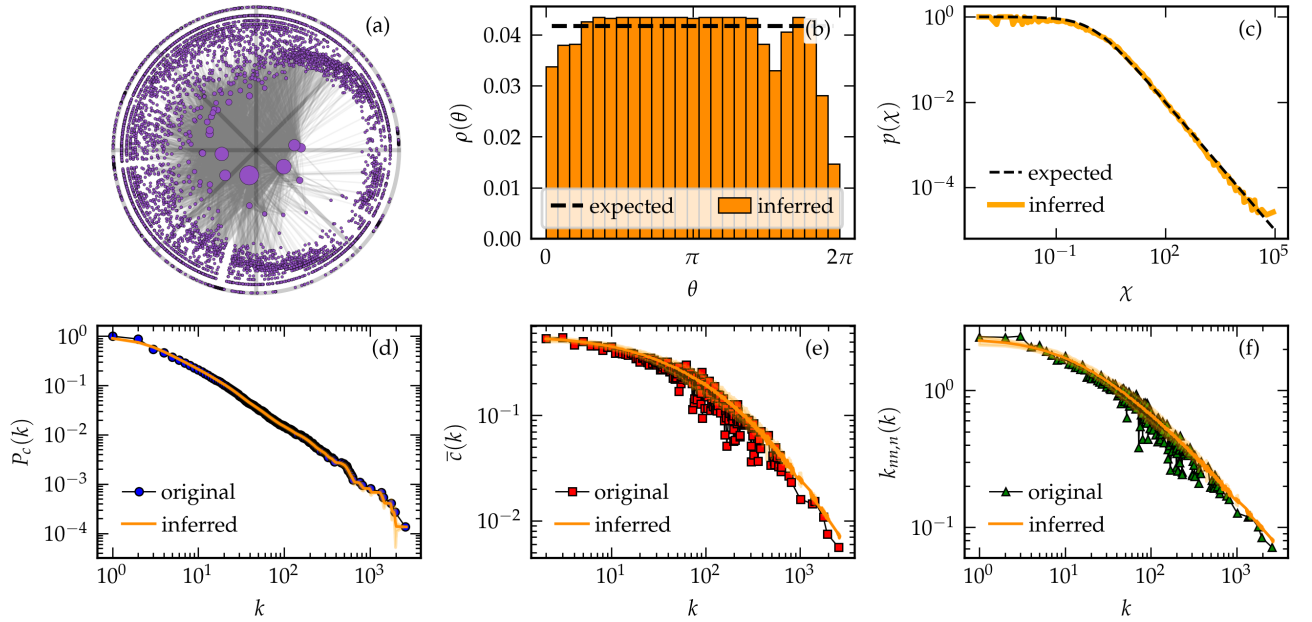


Figure S3: Summary of the results of Mercator for the WordAdjacency-English network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 10% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

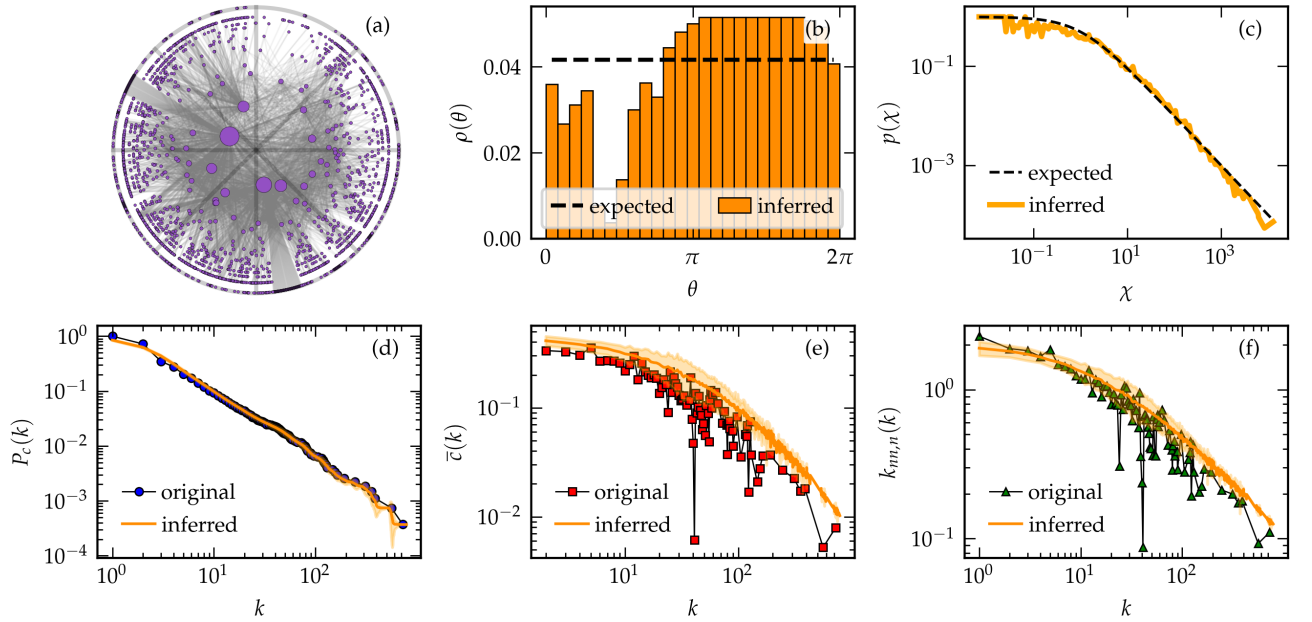


Figure S4: Summary of the results of Mercator for the WordAdjacency–Japanese network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 50% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

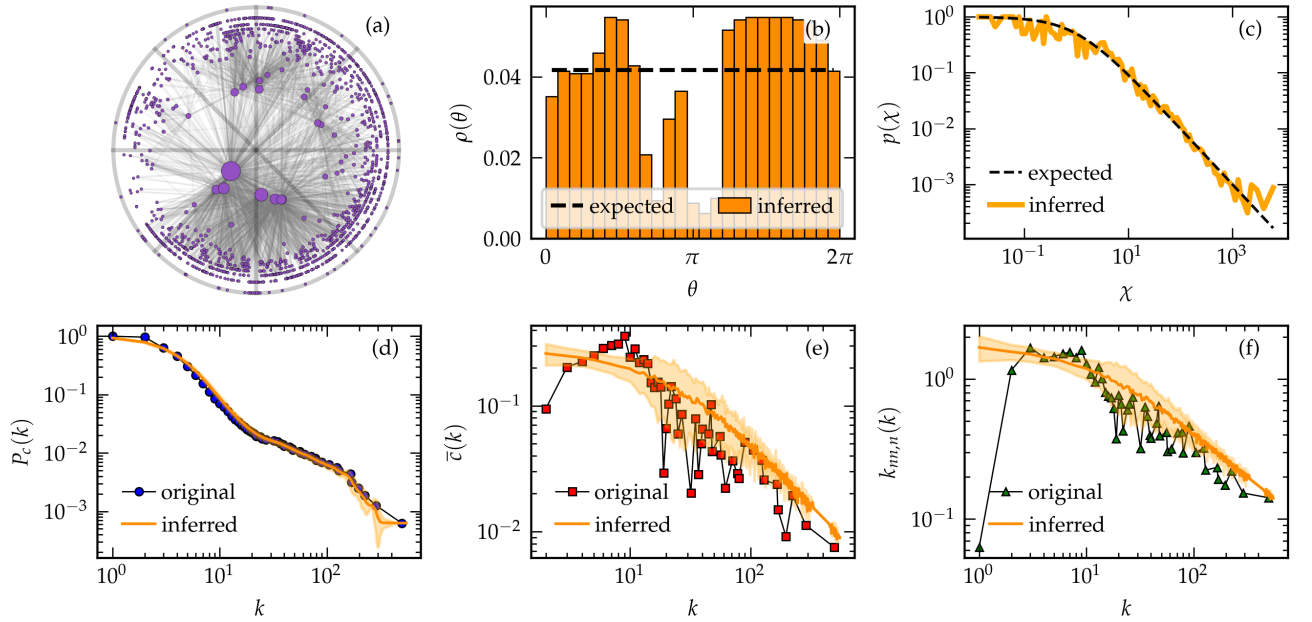


Figure S5: Summary of the results of Mercator for the MB-R.norvegicus network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 50% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

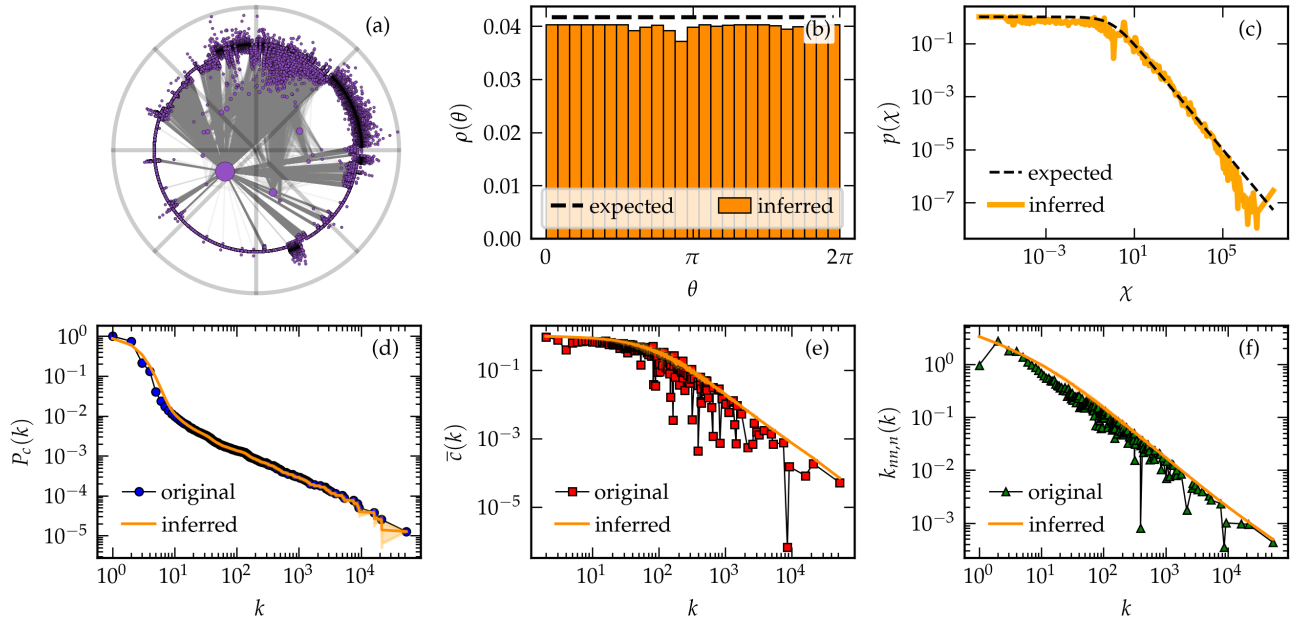


Figure S6: Summary of the results of Mercator for the WikiTalk–Catalan network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 10% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

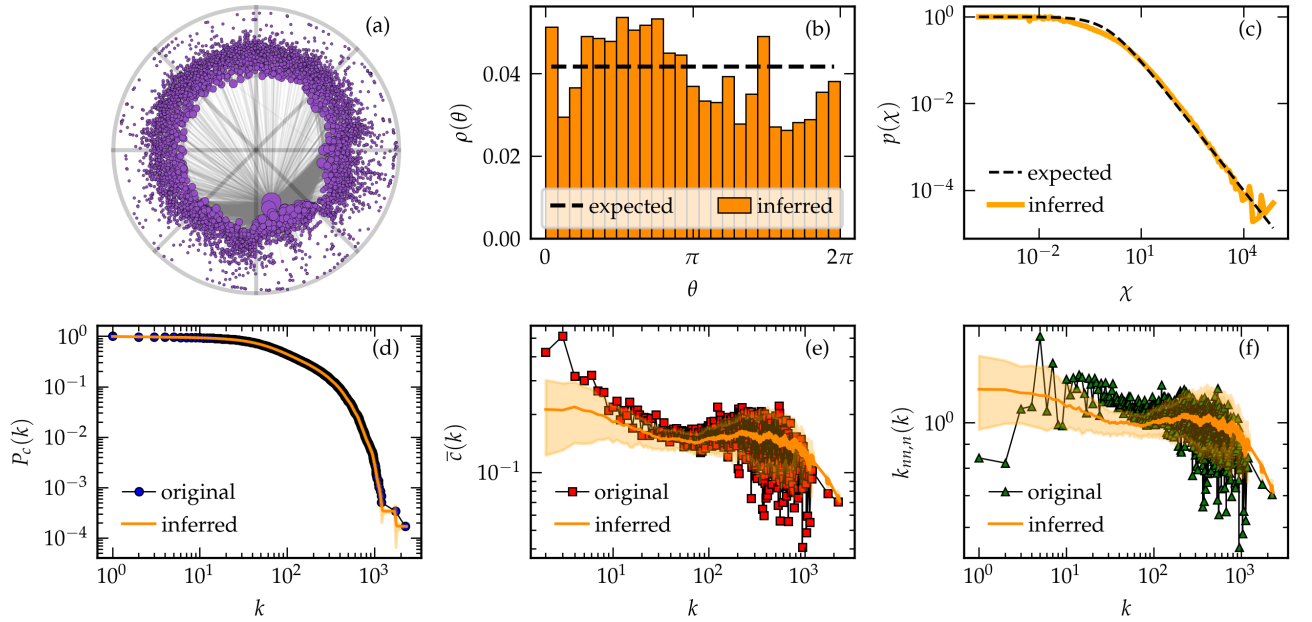


Figure S7: Summary of the results of Mercator for the GI-S.cerevisiae network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 1% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

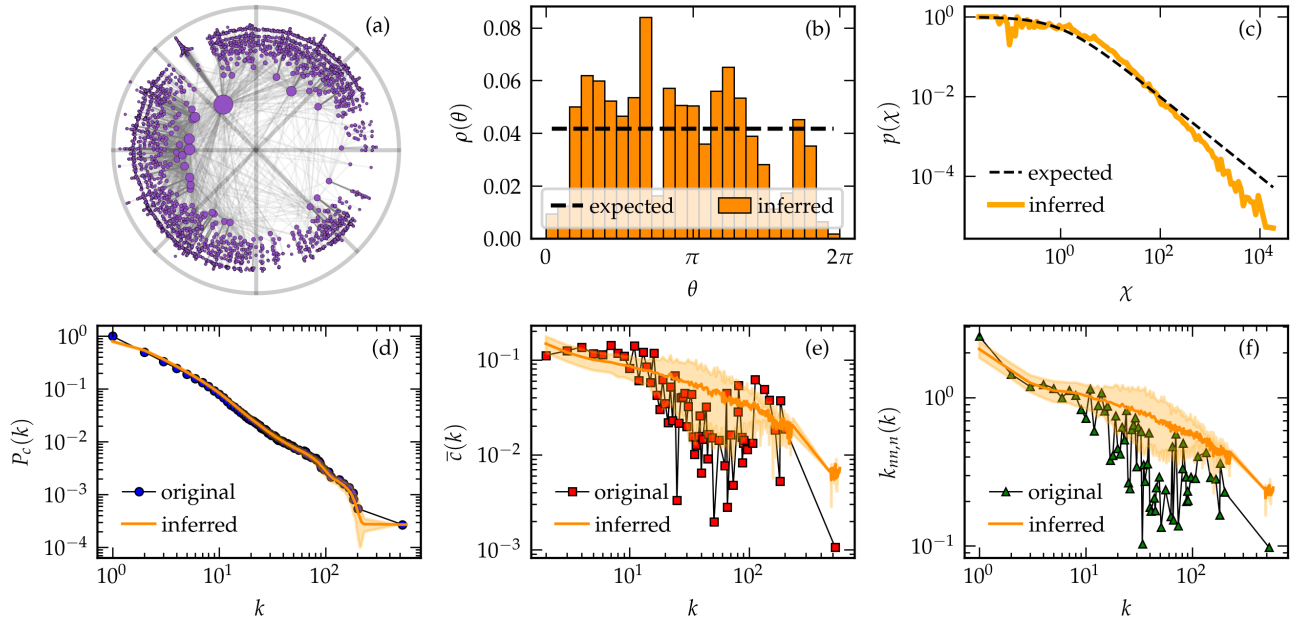


Figure S8: Summary of the results of Mercator for the GMP-C.elegans network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 50% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

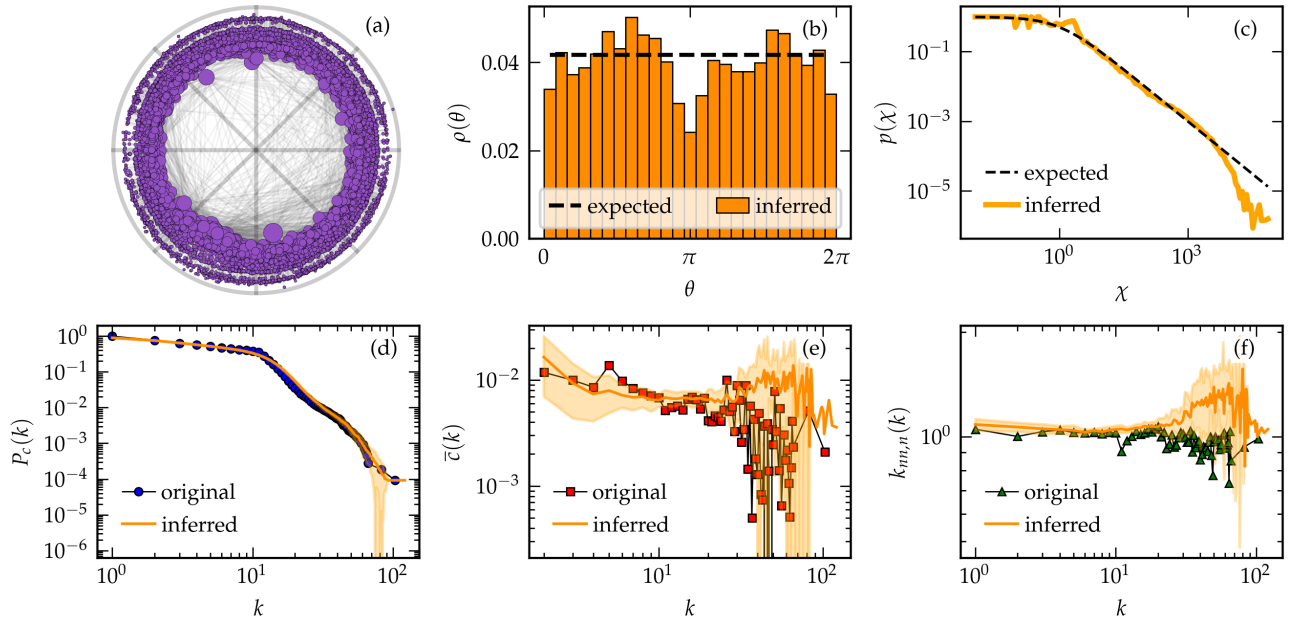


Figure S9: Summary of the results of Mercator for the Gnutella network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 40% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

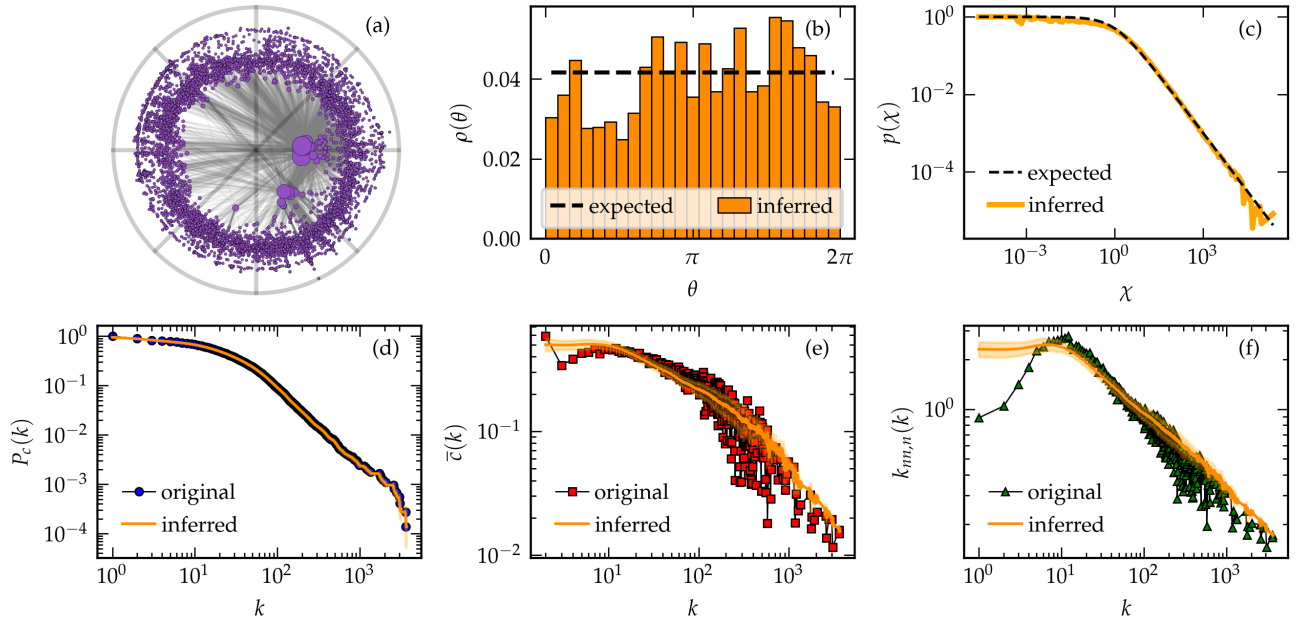


Figure S10: Summary of the results of Mercator for the PPI–*S.cerevisiae* network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 3% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

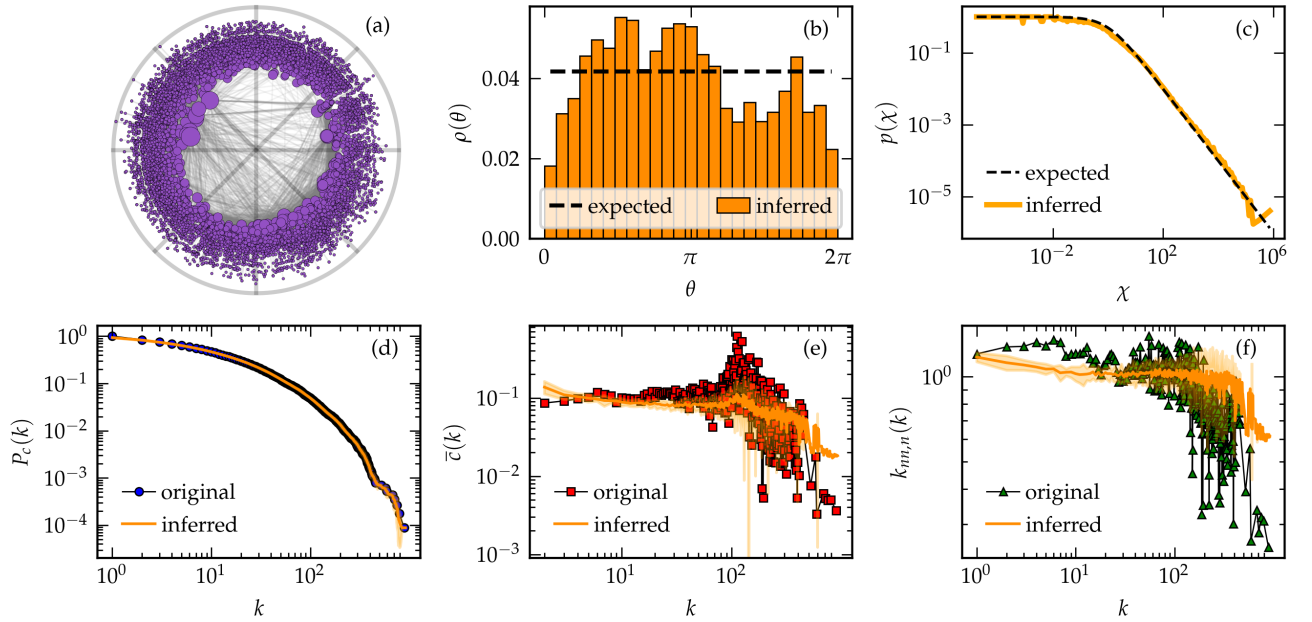


Figure S11: Summary of the results of Mercator for the PPI-D.melanogaster network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 20% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

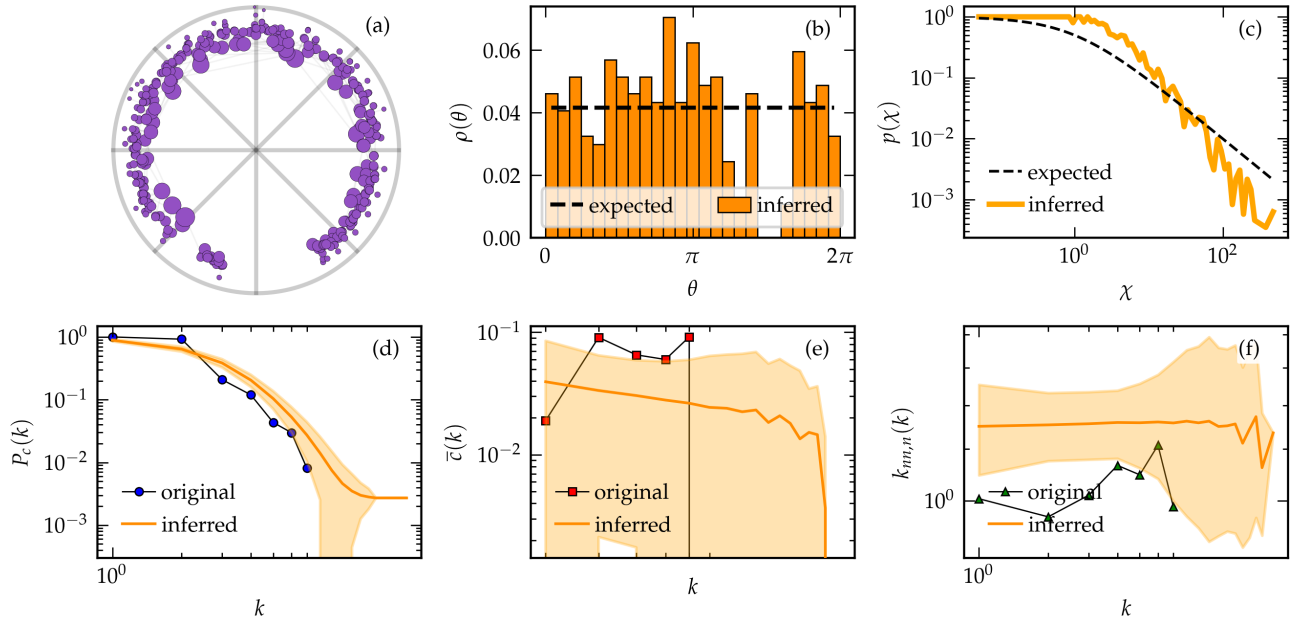


Figure S12: Summary of the results of Mercator for the Transport–London network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 100% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

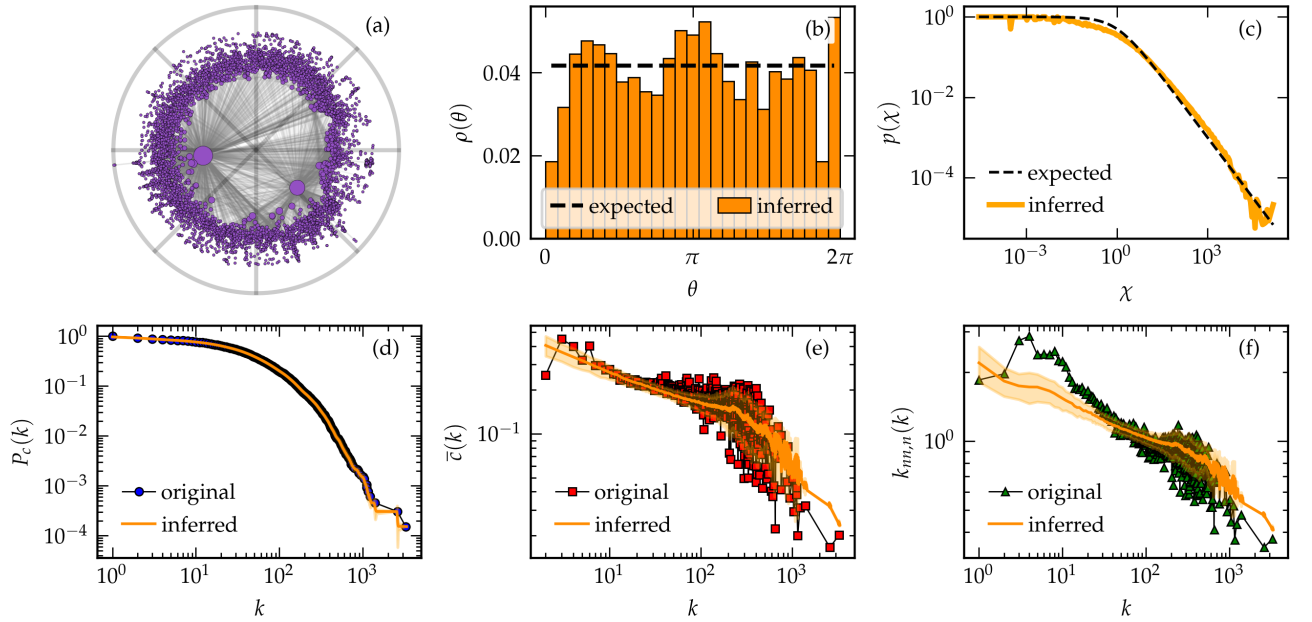


Figure S13: Summary of the results of Mercator for the GMP–*S.cerevisiae* network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 5% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

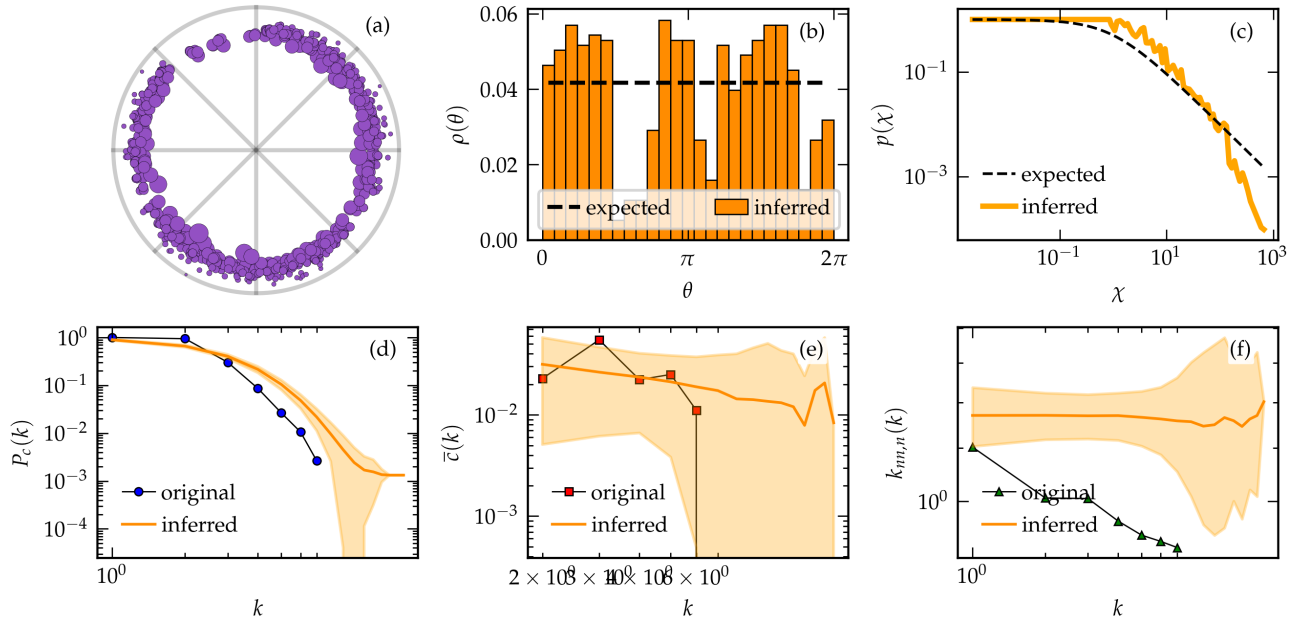


Figure S14: Summary of the results of Mercator for the Internet-PoP network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 100% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

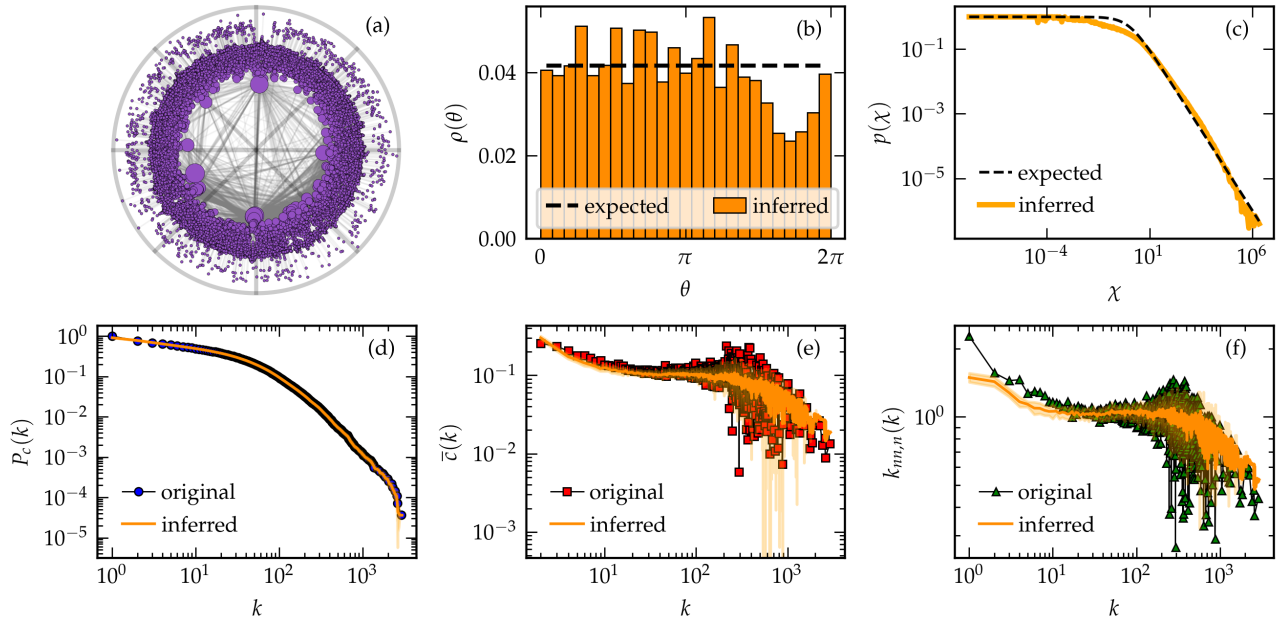


Figure S15: Summary of the results of Mercator for the PPI-H.sapiens network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 5% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

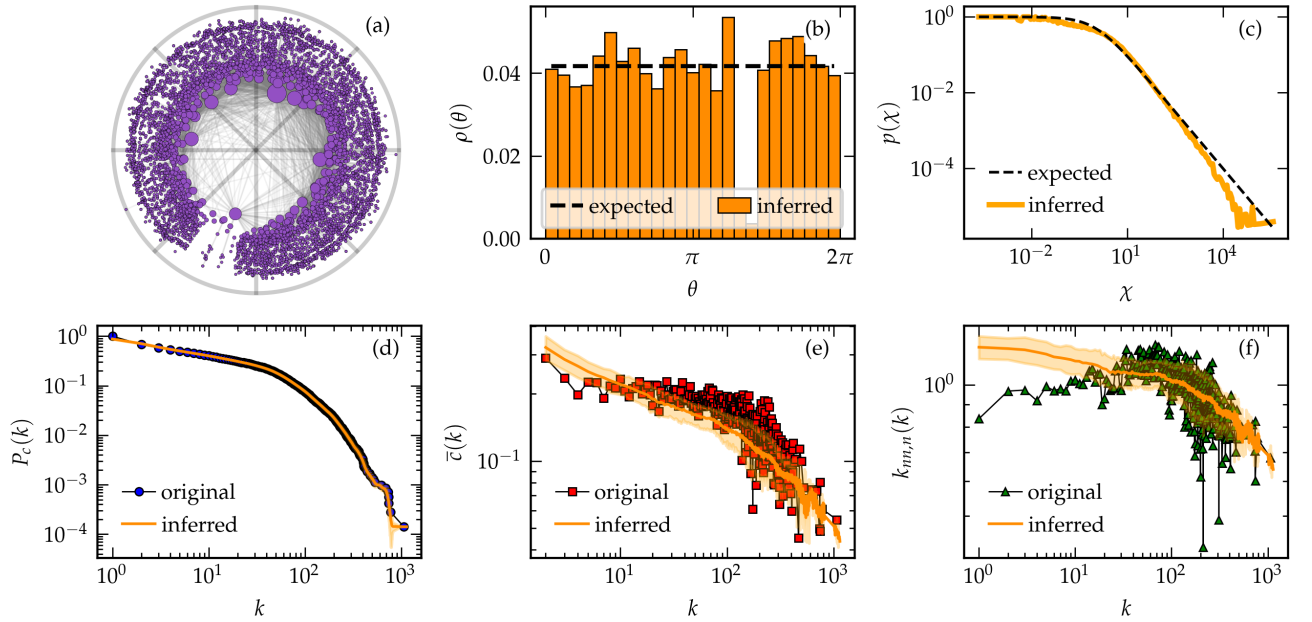


Figure S16: Summary of the results of Mercator for the WikiVote network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 10% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.

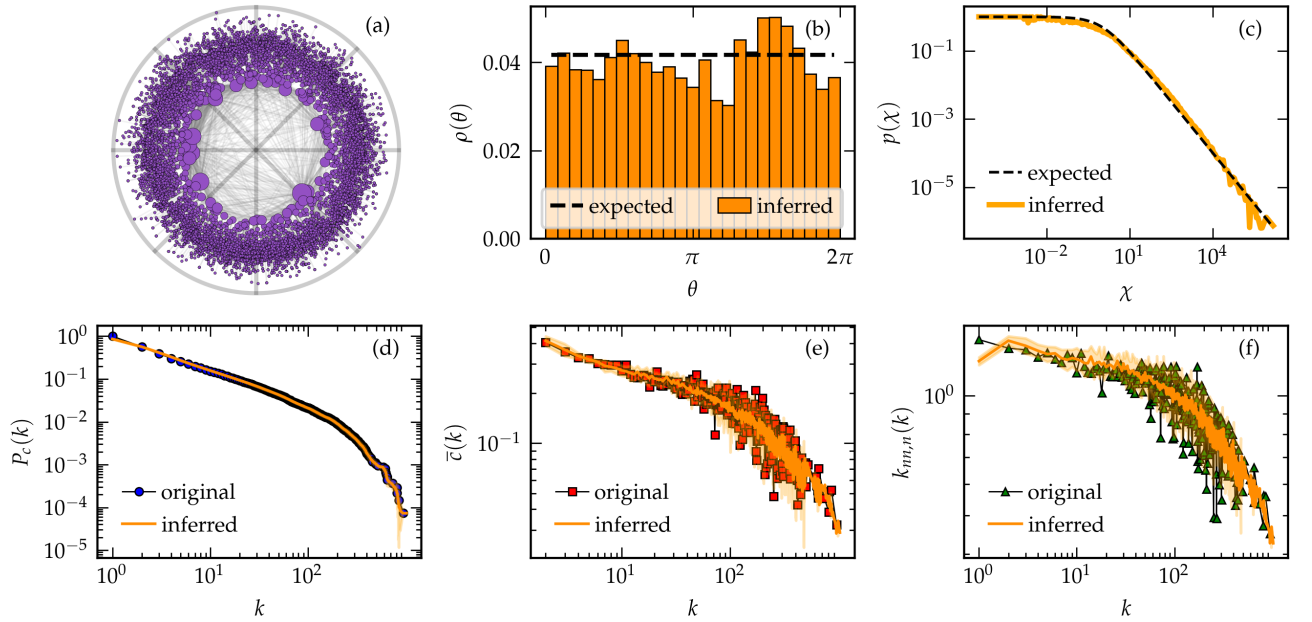


Figure S17: Summary of the results of Mercator for the MathOverflow network. **(a)** Representation of the embedding in the hyperbolic plane as defined by the $\mathbb{H}^2$ -model. The top 5% most geometric edges are shown. **(b)** Comparison between the expected and inferred densities of nodes along the circle. **(c)** Comparison between the probability distribution as expected based on the model (expected) as well as the actual distribution based on the inferred coordinates (inferred). The reproduction of the topological properties is also given: **(d)** the complementary cumulative degree distribution, **(e)** the average local clustering coefficient per degree class and **(f)** the degree-degree correlations per degree class. The inferred results are obtained by generating 100 realizations of the $\mathbb{S}^1$ -model based on the inferred coordinates. The orange shaded regions represent the 2σ confidence interval.