Supplementary Information for
Emergence of geometric Turing patterns in complex networks

Jasper van der Kolk,[1,2,*] Guillermo García-Pérez,[3] Nikos E. Kouvaris,[4]
M. Ángeles Serrano,[1,2,5,†] and Marián Boguñá[1,2,‡]

[1] *Departament de Física de la Matèria Condensada, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain*

[2] *Universitat de Barcelona Institute of Complex Systems (UBICS), Barcelona, Spain*

[3] *Algorithmiq Ltd, Kanavakatu 3C, FI-00160 Helsinki, Finland*

[4] *Dribia Data Research S. L., Barcelona, Spain*

[5] *Institució Catalana de Recerca i Estudis Avançats (ICREA), Passeig Lluís Companys 23, E-08010 Barcelona, Spain*

CONTENTS

* jasper.vanderkolk@ub.edu
† marian.serrano@ub.edu
‡ marian.boguna@ub.edu

S1. THE γ DEPENDENCE OF $\hat{r}$

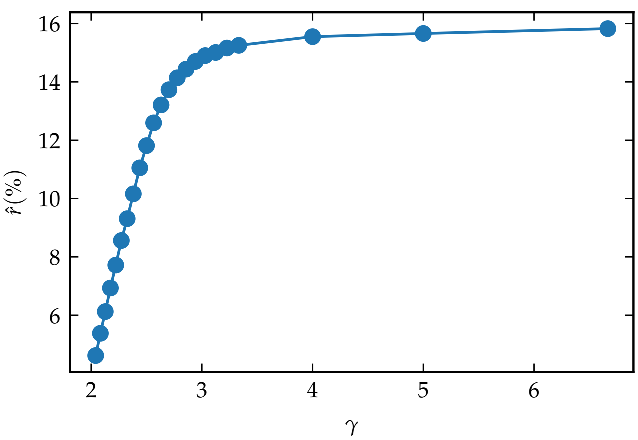
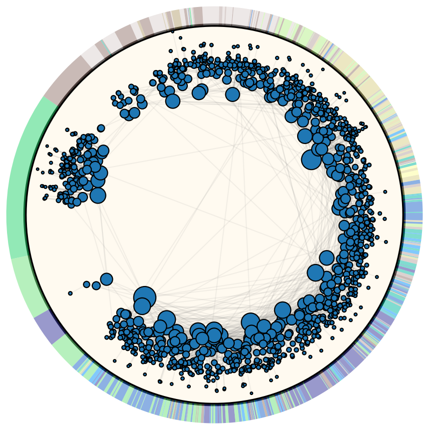


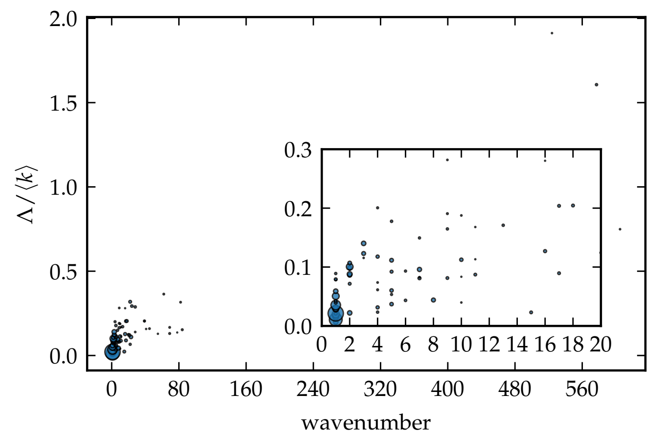
FIG. S1: The dependence of the amount of eigenvectors classified as periodic (at $p = 0.01$) on the heterogeneity of the network.

S2. REAL NETWORKS

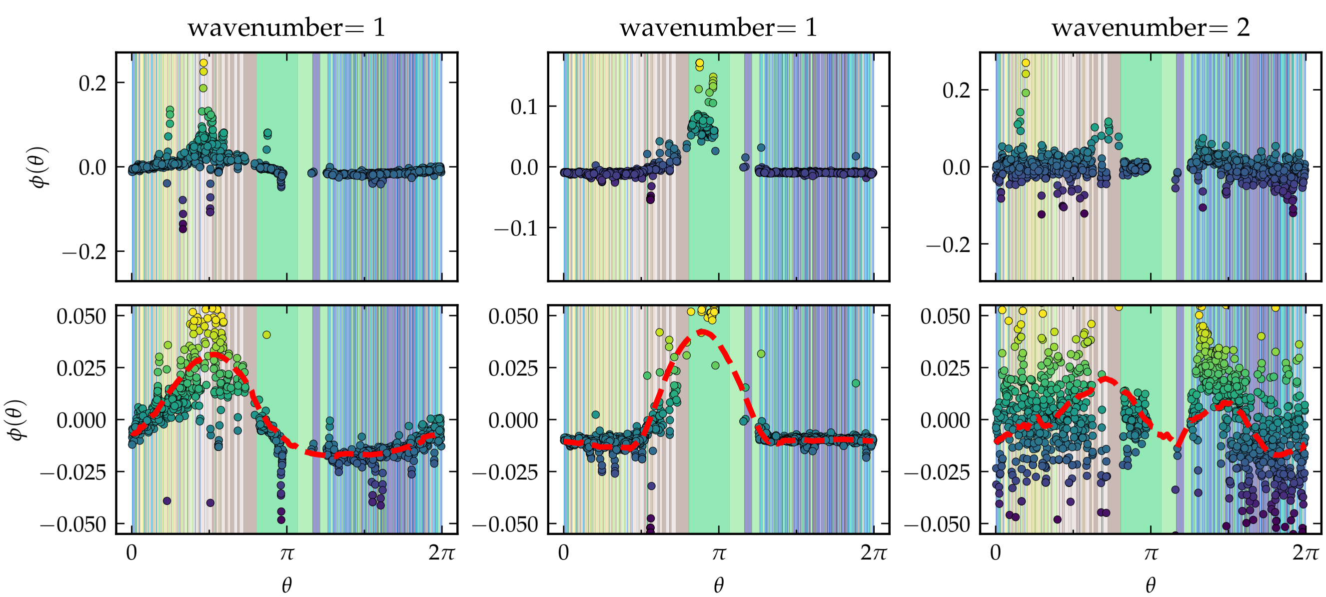
A. Airports



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



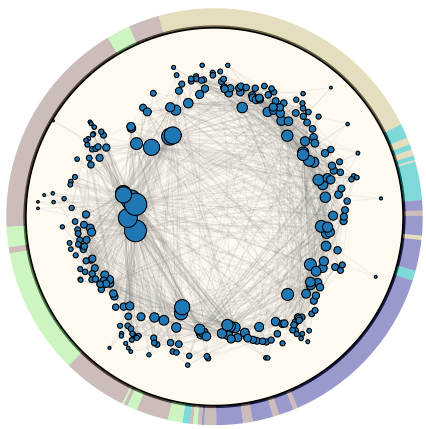
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber. The inset is an enhancement for small wavenumbers.



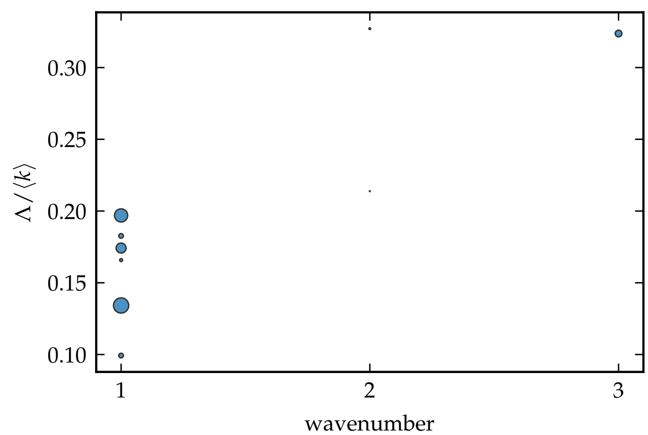
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S2: Network analysis for the network Airport, with network parameters $N = 1226$, $\langle k \rangle = 3.9$, $\beta = 1.0$ and 14 modules which leads to a modularity of 0.7.

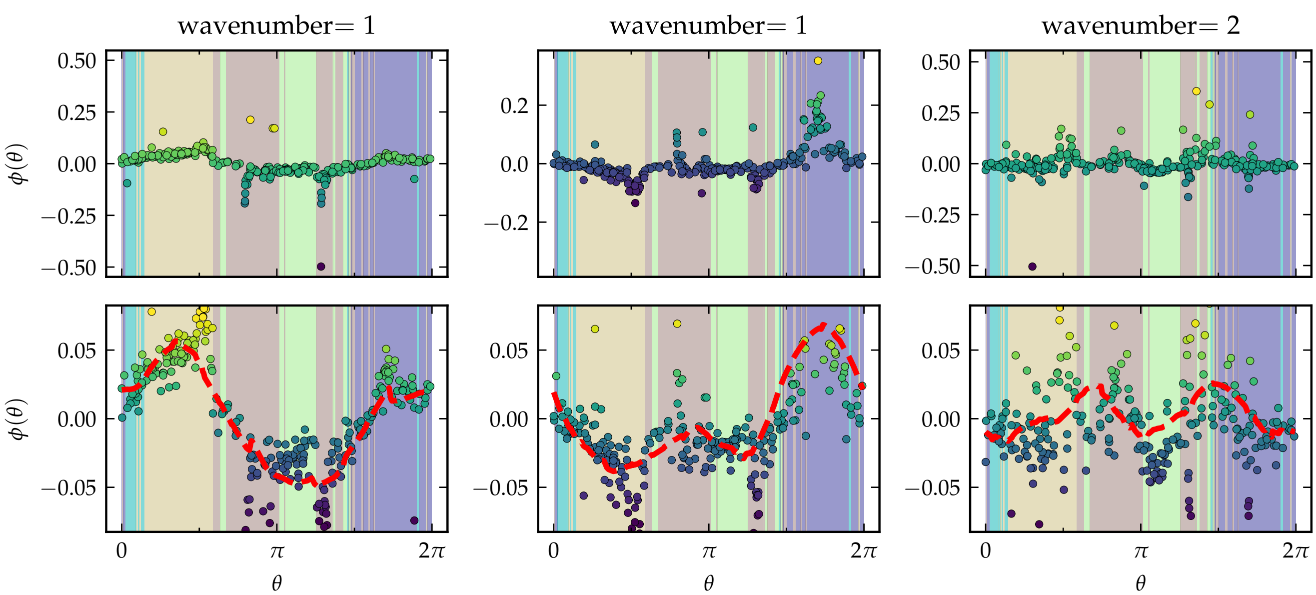
B. CElegans-C



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



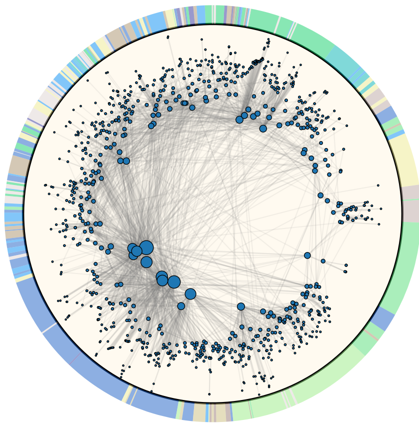
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber.



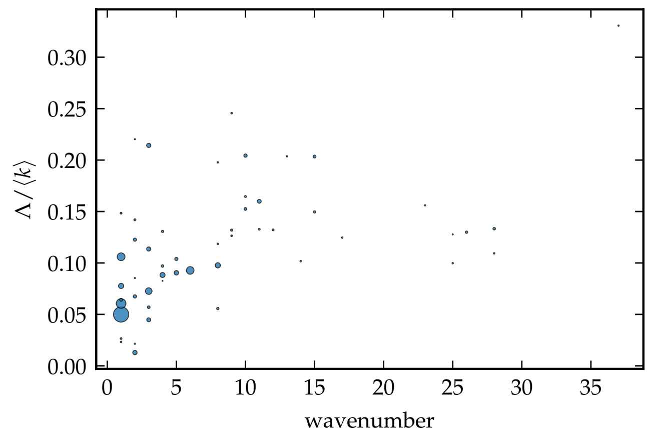
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S3: Network analysis for the network CElegans-C, with network parameters $N = 279$, $\langle k\rangle = 16.4$, $\beta = 1.5$ and 5 modules which leads to a modularity of 0.4.

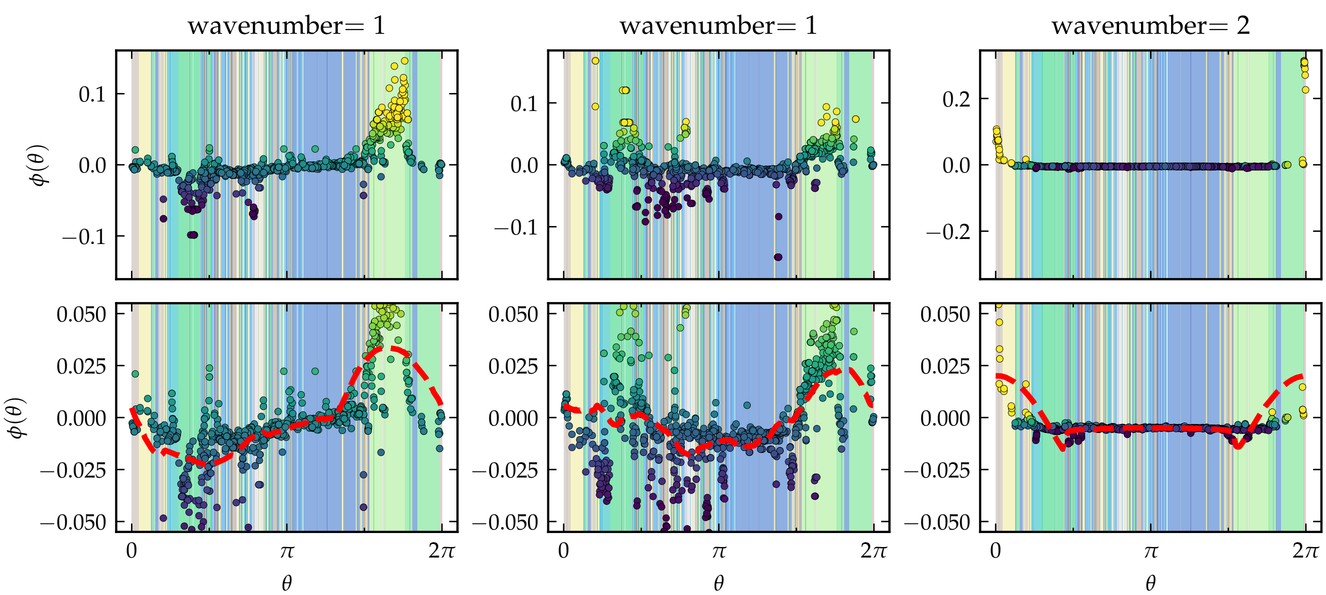
C. CElegans-G



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



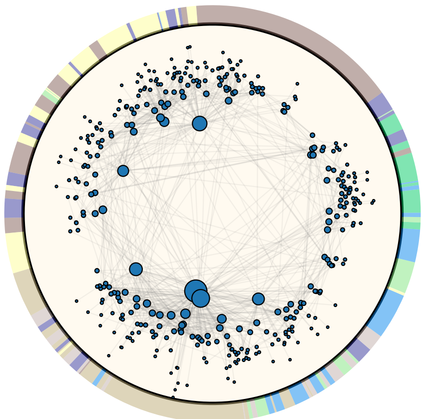
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber.



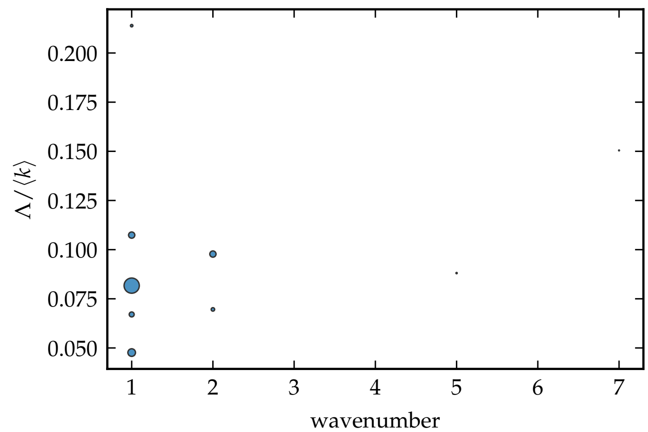
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S4: Network analysis for the network CElegans-G, with network parameters $N = 878$, $\langle k\rangle = 7.2$, $\beta = 2.6$ and 15 modules which leads to a modularity of 0.6.

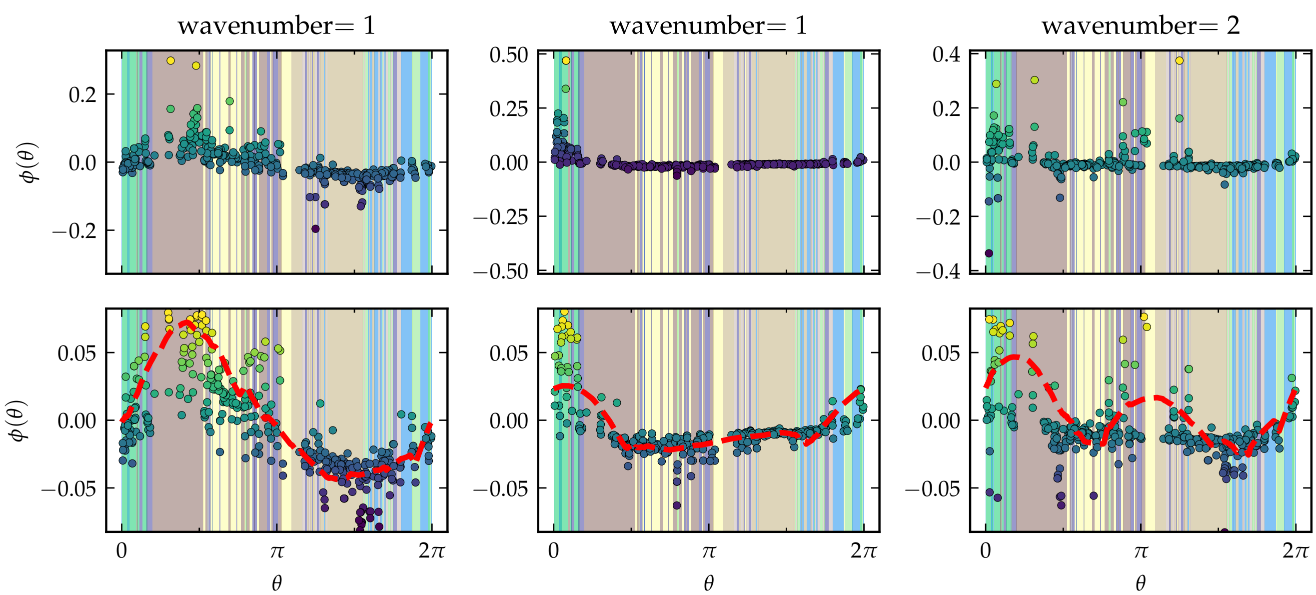
D. Commodities



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



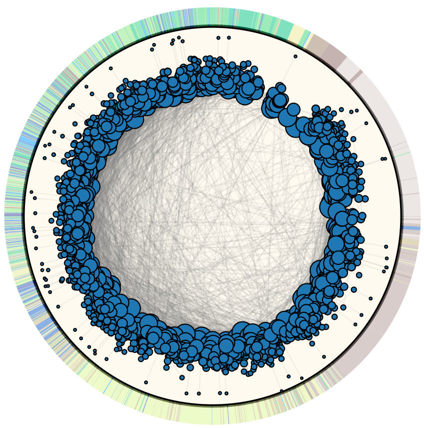
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber.



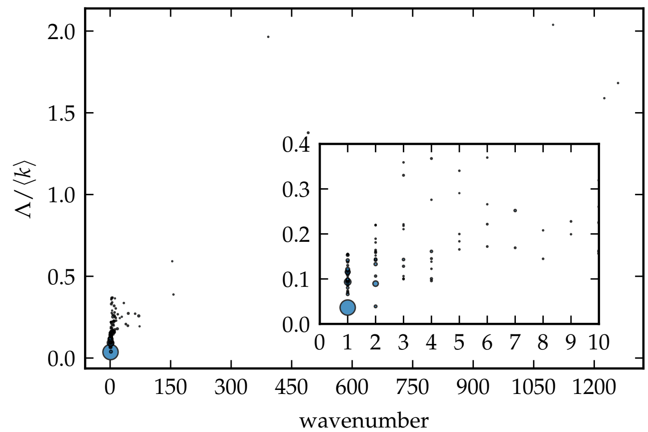
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S5: Network analysis for the network Commodities, with network parameters $N = 374$, $\langle k\rangle = 5.8$, $\beta = 1.1$ and 8 modules which leads to a modularity of 0.6.

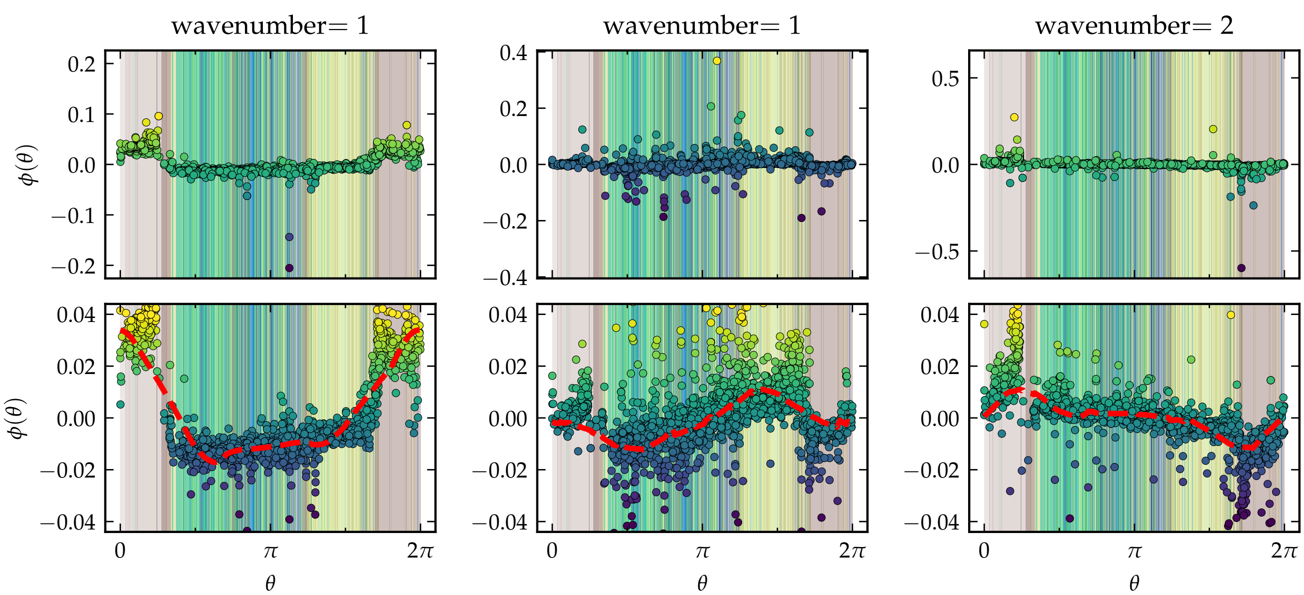
E. FriendsOFF



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



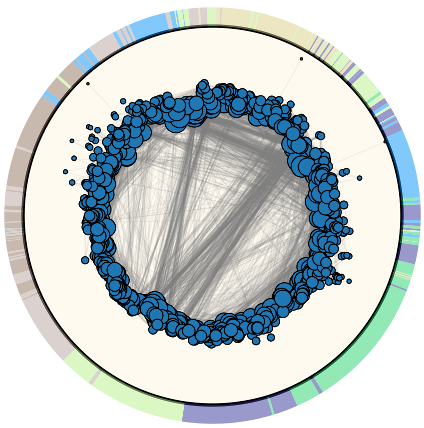
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber. The inset is an enhancement for small wavenumbers.



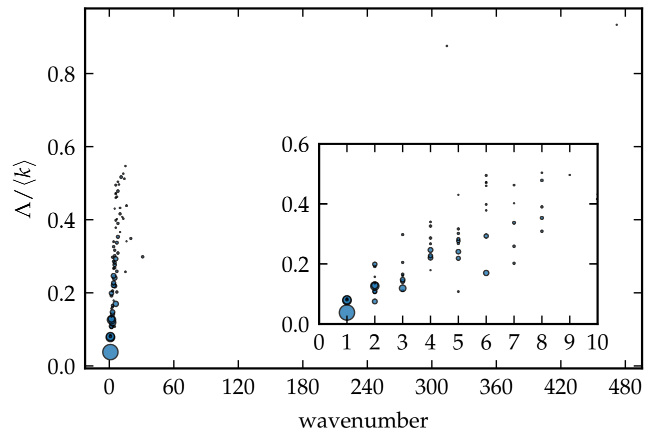
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S6: Network analysis for the network FriendsOFF, with network parameters $N = 2539$, $\langle k\rangle = 8.2$, $\beta = 1.3$ and 13 modules which leads to a modularity of 0.6.

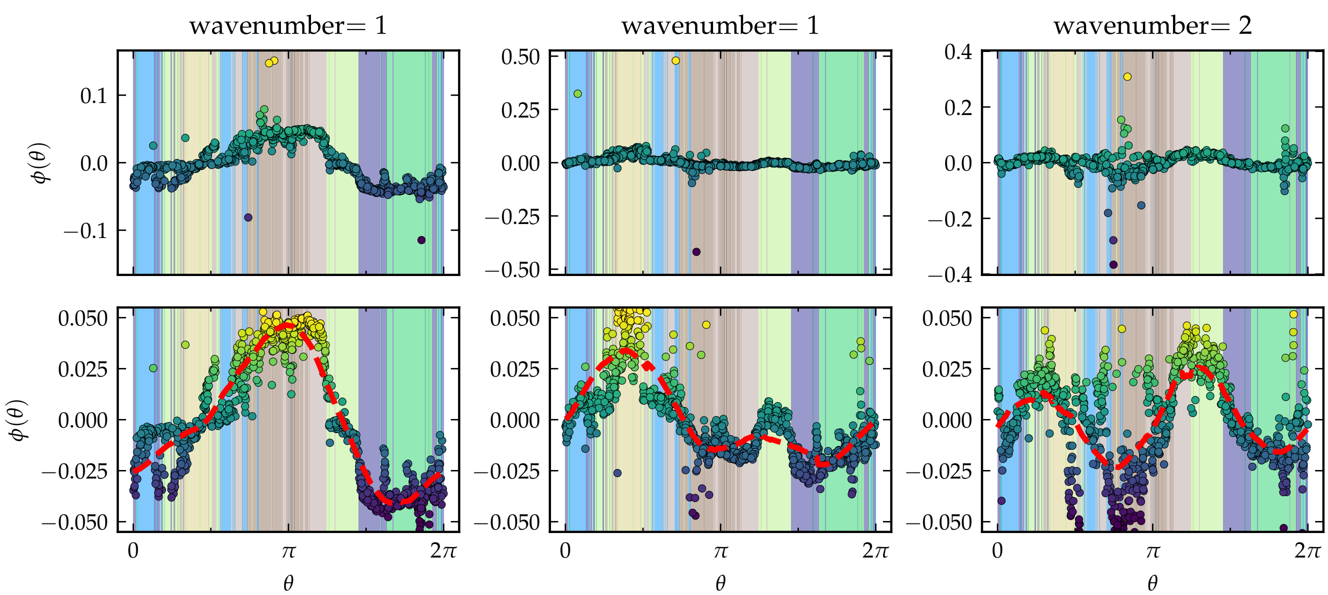
F. Human-C



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



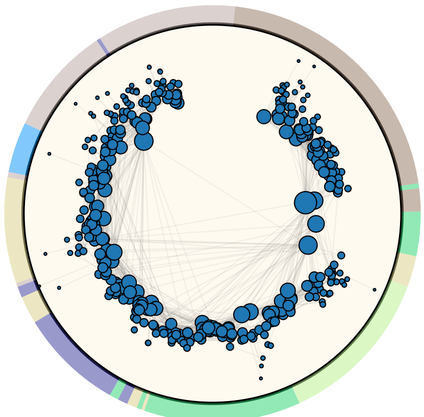
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber. The inset is an enhancement for small wavenumbers.



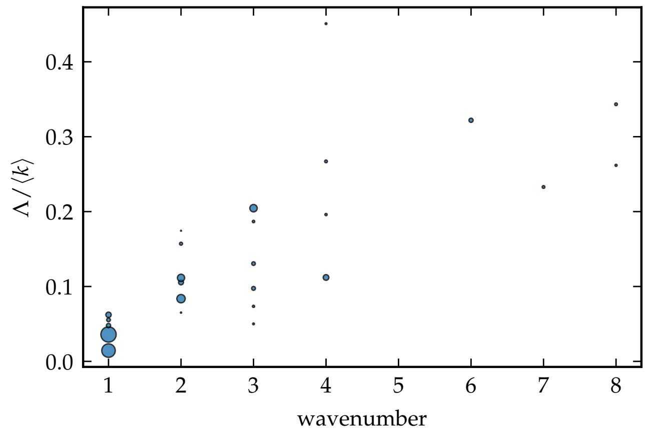
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S7: Network analysis for the network Human-C, with network parameters $N = 989$, $\langle k\rangle = 36.1$, $\beta = 2.3$ and 7 modules which leads to a modularity of 0.6.

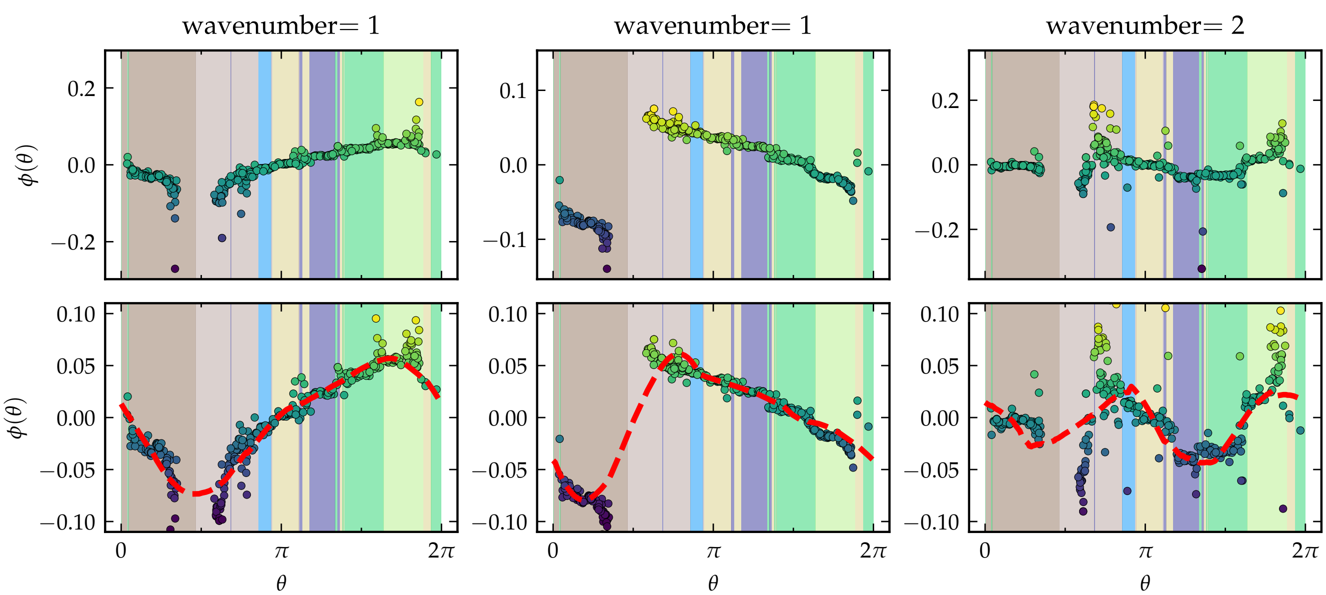
G. List Contacts



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



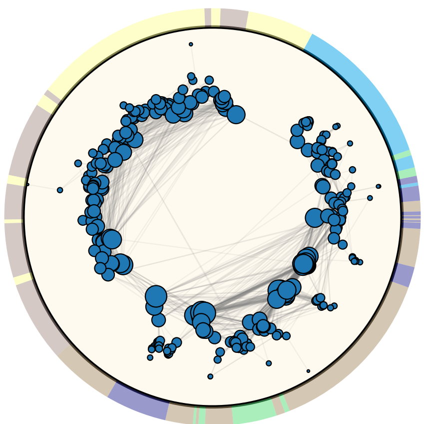
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber.



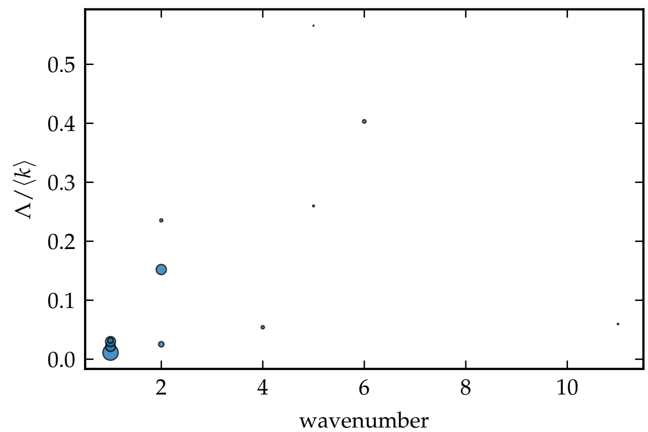
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S8: Network analysis for the network List Contacts, with network parameters $N = 410$, $\langle k\rangle = 13.5$, $\beta = 2.2$ and 7 modules which leads to a modularity of 0.7.

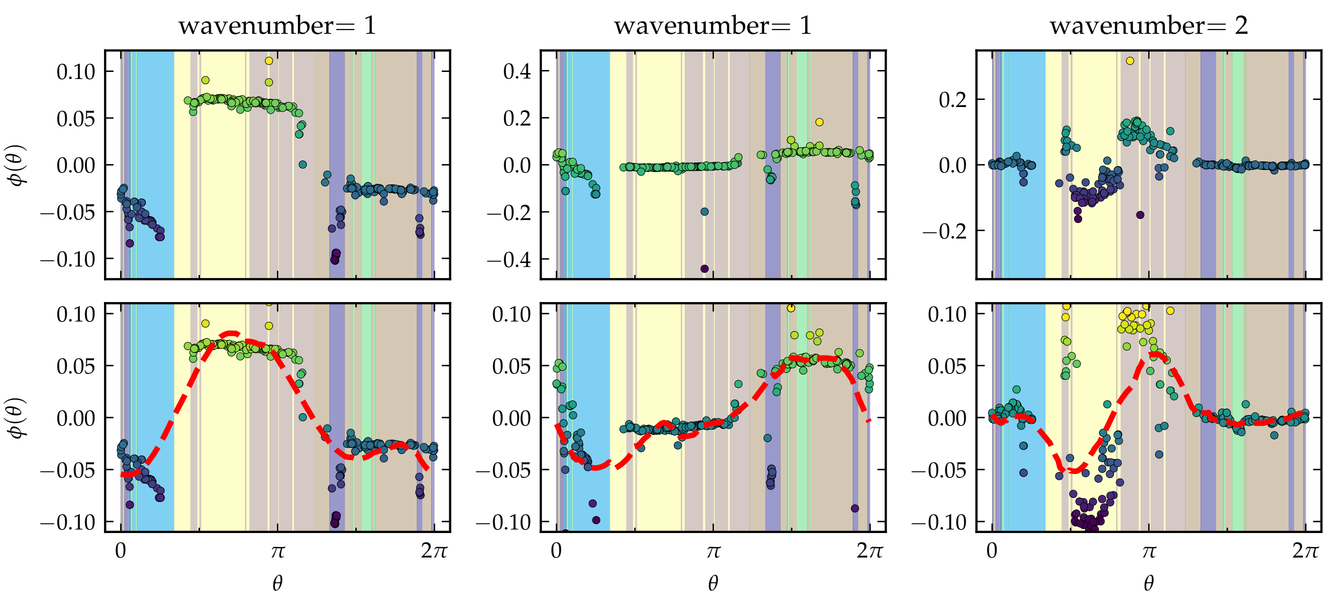
H. Malaria-G



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



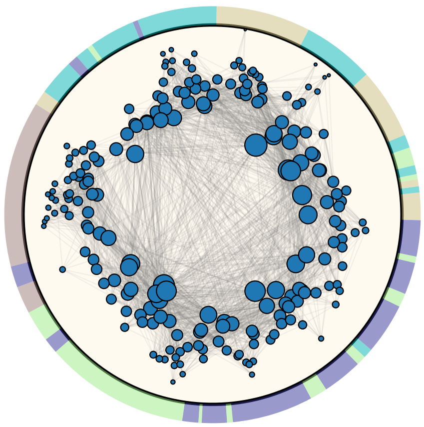
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber.



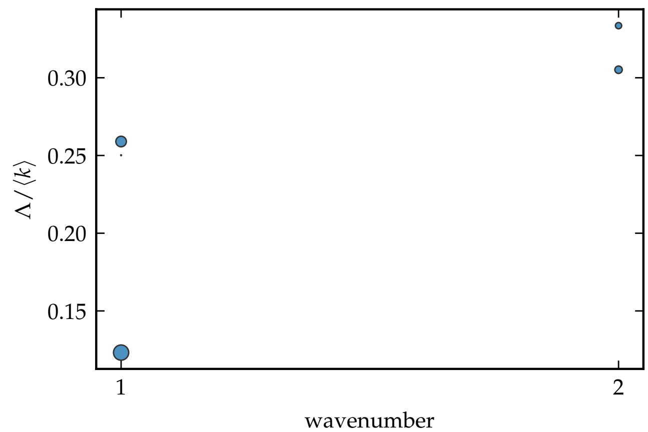
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S9: Network analysis for the network Malaria-G, with network parameters $N = 307$, $\langle k\rangle = 18.3$, $\beta = 2.9$ and 6 modules which leads to a modularity of 0.6.

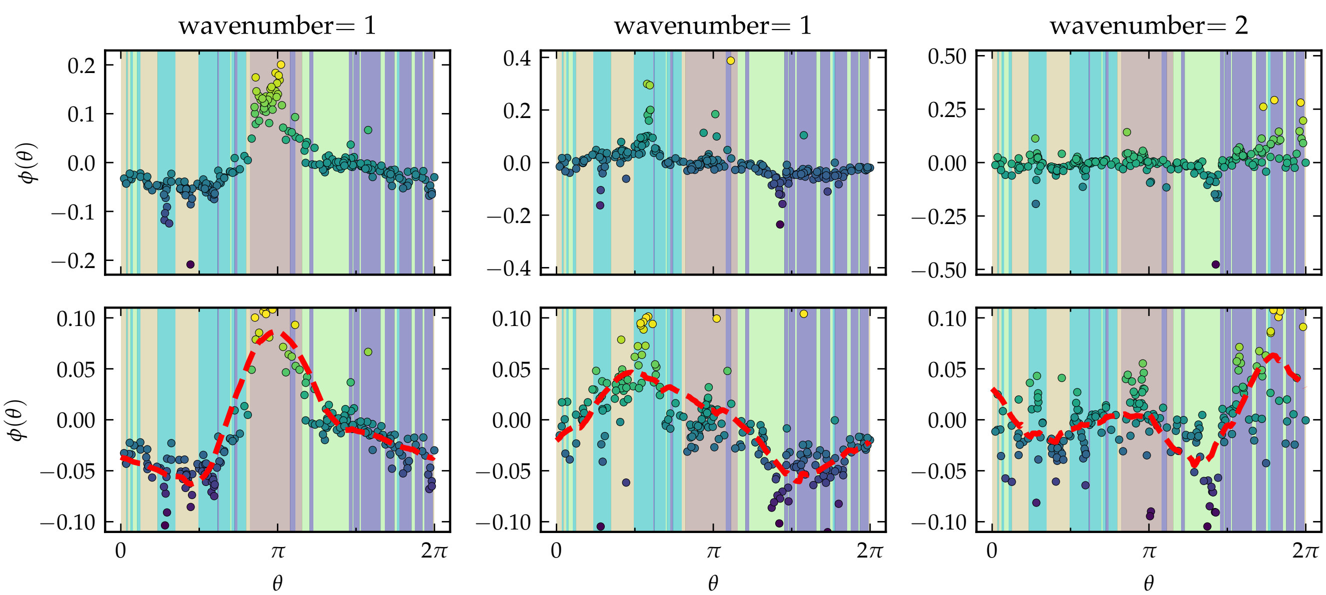
I. Mouse-C



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



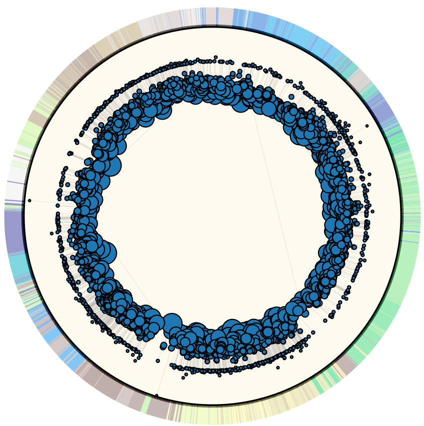
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber.



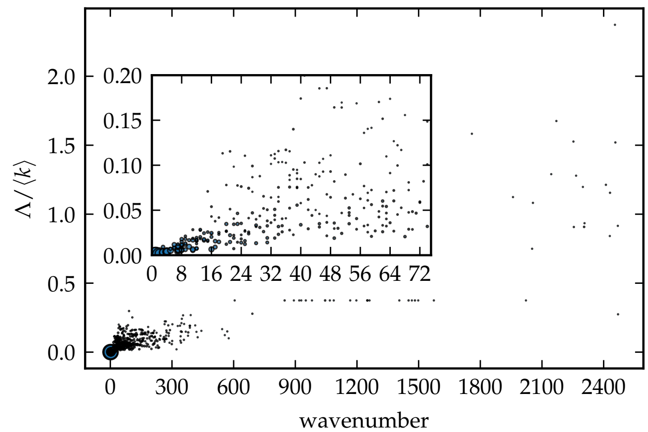
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S10: Network analysis for the network Mouse-C, with network parameters $N = 213$, $\langle k \rangle = 27.9$, $\beta = 2.0$ and 5 modules which leads to a modularity of 0.4.

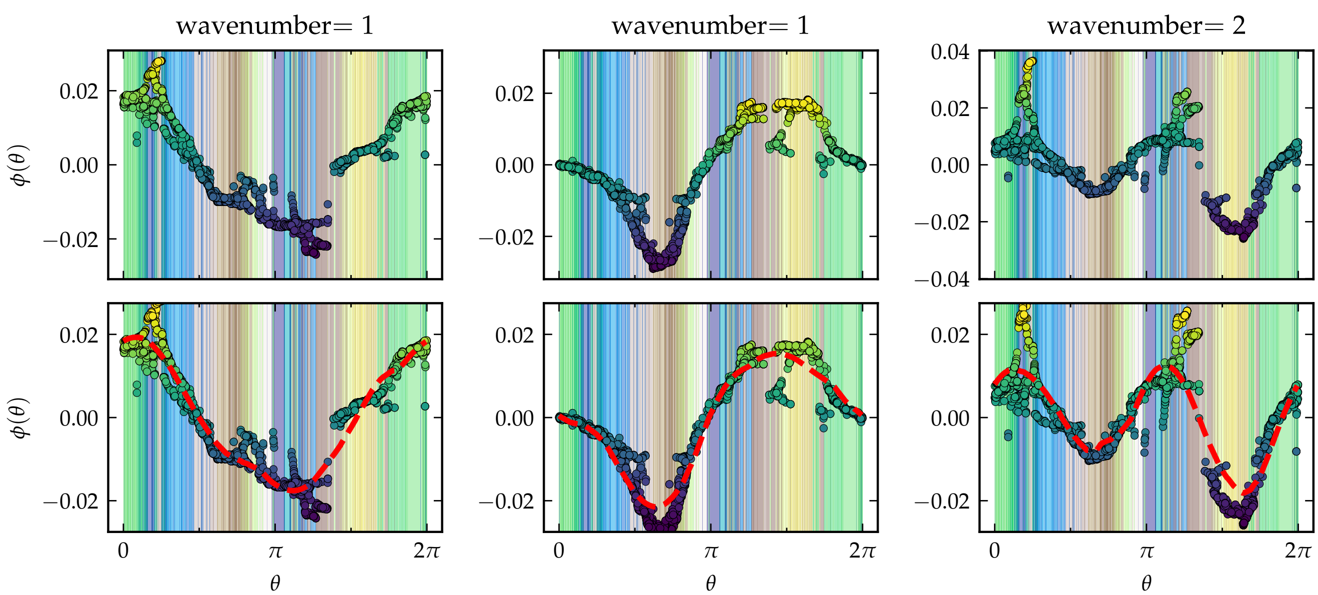
J. Power



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



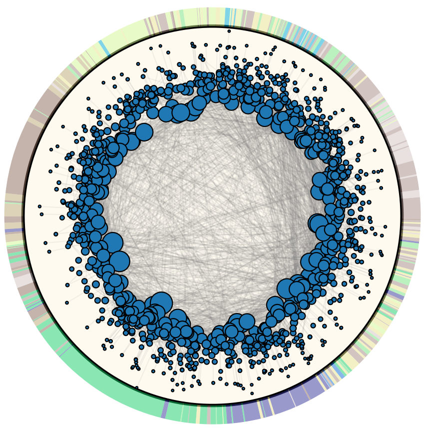
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber. The inset is an enhancement for small wavenumbers.



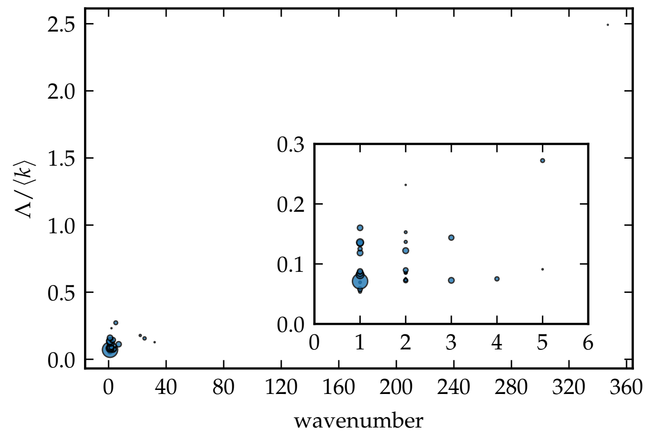
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S11: Network analysis for the network Power, with network parameters $N = 4941$, $\langle k\rangle = 2.7$, $\beta = 1.3$ and 36 modules which leads to a modularity of 0.9.

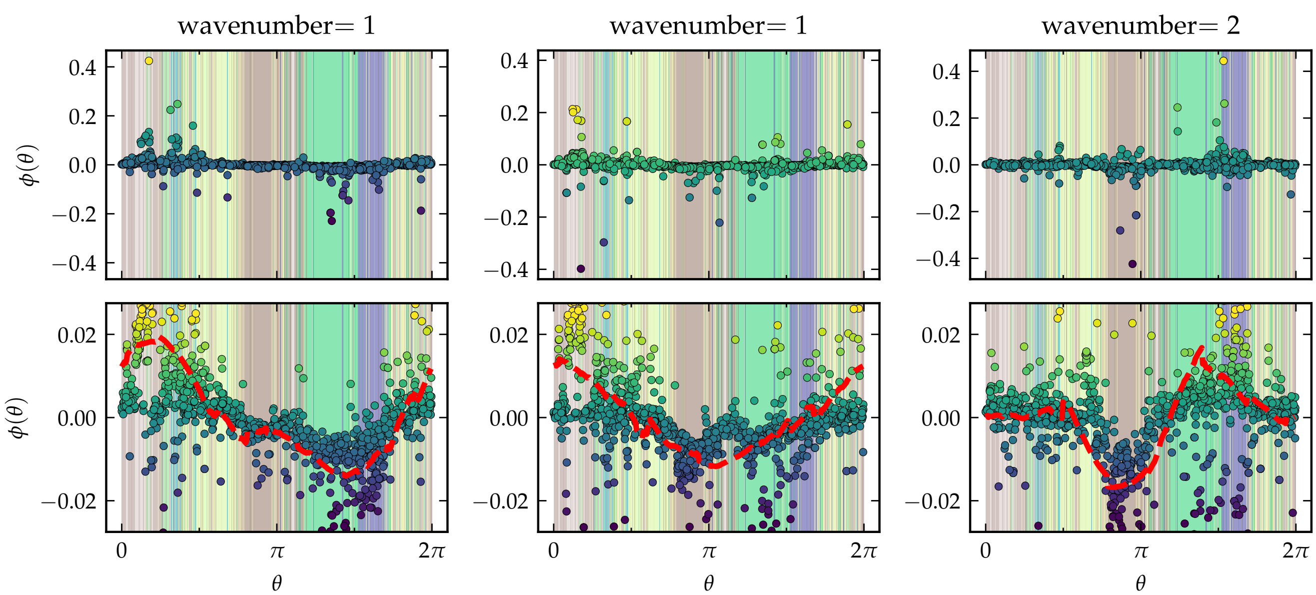
K. URV-Email



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



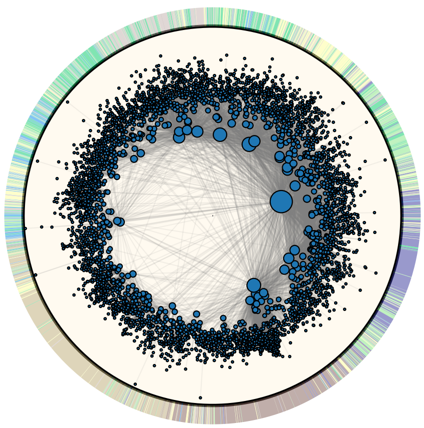
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber. The inset is an enhancement for small wavenumbers.



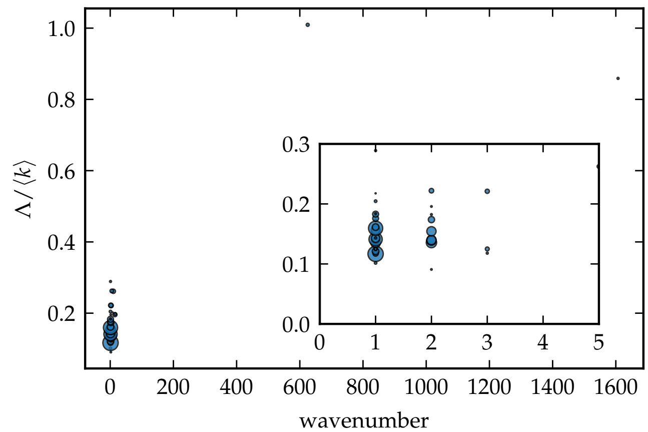
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S12: Network analysis for the network URV-Email, with network parameters $N = 1133$, $\langle k\rangle = 9.6$, $\beta = 1.36$ and 11 modules which leads to a modularity of 0.6.

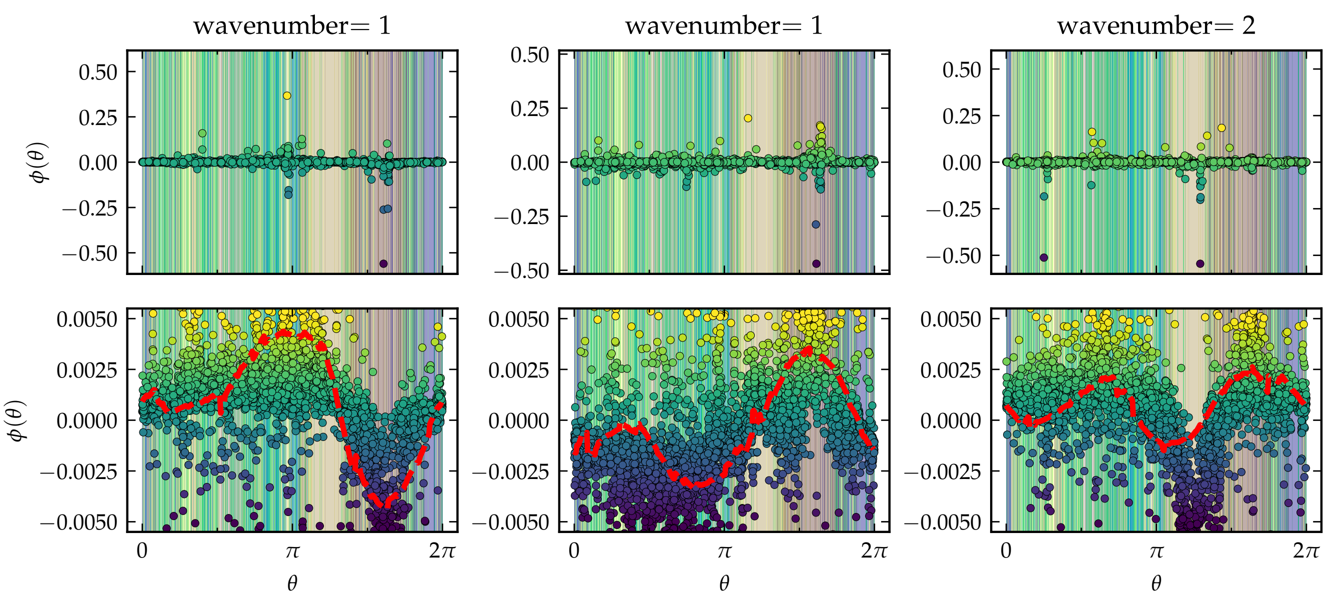
L. Wikipedia



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



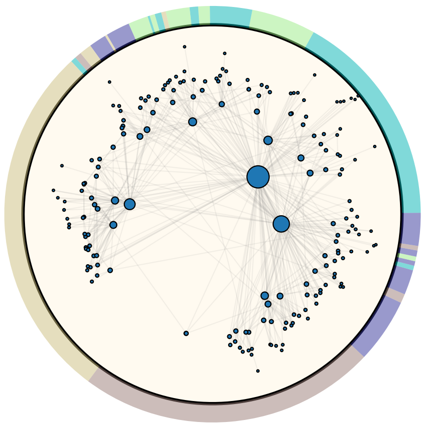
(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber. The inset is an enhancement for small wavenumbers.



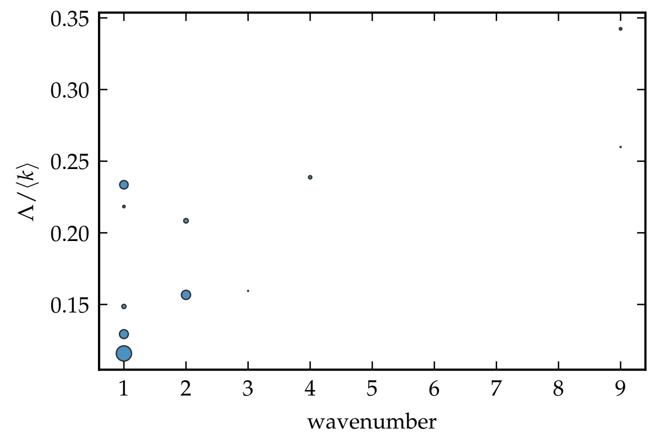
(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S13: Network analysis for the network Wikipedia, with network parameters $N = 4589$, $\langle k\rangle = 46.4$, $\beta = 1.2$ and 8 modules which leads to a modularity of 0.4.

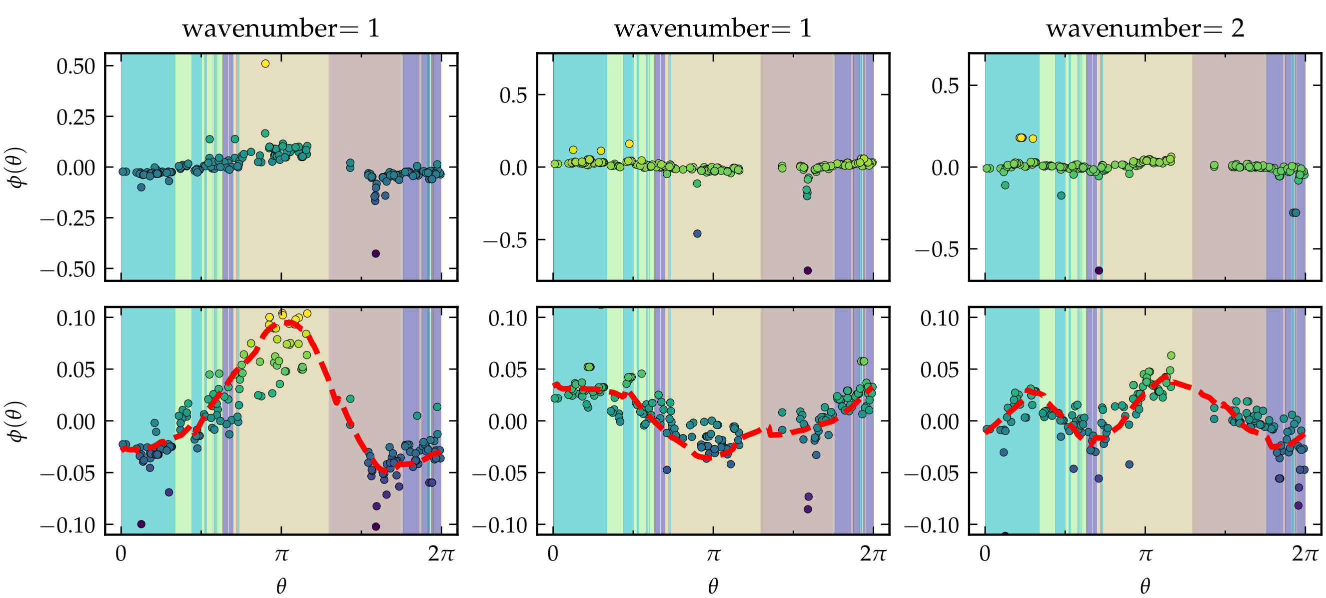
M. WTW-2013



(a) Hyperbolic representation of the network embedding. The node size is proportional to the hidden degree, and the colors on the outer arc of the circle represent the communities.



(b) The dispersion relation of the network Laplacian. For each eigenvector the wavenumber is determined by taking the largest peak of the Fourier spectrum. The size of the points is proportional to the size of the peak and only peaks that are significant with respect to the spectrum of white noise are kept. The eigenvalue of the eigenvector is then plotted against the wavenumber.



(c) Eigenvectors as a function of the angular coordinate of the nodes. The different rows represent different eigenvectors, where the first and second columns show the most and second most significant patterns with wavenumber = 1 and the last column shows the most significant pattern with wavenumber= 2. The top row shows the entire pattern, where the bottom row zooms in to better observe the pattern. In this row the trendline is also shown, which was obtained by applying the Savitzky-Golay filter with windowsize 0.3π. The background colors represent the communities as found by the Louvain routine.

FIG. S14: Network analysis for the network WTW-2013, with network parameters $N = 189$, $\langle k \rangle = 5.8$, $\beta = 1.9$ and 5 modules which leads to a modularity of 0.5.