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The resonator problem in a spherical GW detector

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Abstract. This paper is a brief summary of the most relevant features of the solution to the general problem of the coupled motion of a set of resonant transducers and a solid sphere when acted upon by a GW excitation, as recently investigated by us. A remarkably elegant theory emerges out of the analysis, which fully displays the system dynamics for *arbitrary* resonator configurations. The power of the method can be used to consider alternative layouts to the TIGA proposal, the virtues and/or drawbacks of which can then be assessed. A specific new resonator distribution will be presented which takes advantage of the significant cross section of a spherical GW detector at its *second* quadrupole resonance and which is also useful for eventual *monopole* radiation sensing.

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One of the most important problems which has to be addressed in relation to spherical GW detector science is the one of *motion sensing*, that is, how can we actually sort out the antenna's vibrations when an incoming GW impinges on it and how the amplitudes and incidence direction of the wave can then be *deconvolved* in terms of the system readout. An advantageous device to implement such a system is a set of *resonant transducers* attached to the sphere's surface, the motions of which can be followed up by means of suitable electronics. But addition of such a set of resonators *does affect* the sphere's motions, so that a *coupled* dynamics regime is entered by the whole system which has to be studied carefully before reliable conclusions can be drawn. This is by no means a trivial problem, even from a purely theoretical point of view. In this paper we will try to enumerate the fundamental traits of the resonator problem theory, as well as some of the results which derive from the analysis, including a new proposal for a rather complete spherical GW antenna.

The mathematical model for antenna and resonators we shall be using is based on the classical theory of elasticity. We shall, however, omit the details of its implementation as the reader can find a comprehensive description of them and further references in [2]. The main *physical* idea underlying this analysis is that the spherical elastic body will be acted upon by *two* kinds of *external* forces—on the one hand there is the *tidal* force caused by the incoming GW and on the other hand there is the force exerted on it by the set of attached *resonant transducers*. Both are included in the right-hand side of the general equations of motion as *driving terms*, and then a solution to the equations is sought. We make the assumption that N resonators are linked to the sphere's surface which move only *radially*;

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although it is not strictly necessary, we shall also make the simplifying hypothesis that they are all *identical* and that they have a resonance frequency Ω and mass

$$M_{\text{resonator}} = \eta M, \qquad \eta \ll 1 \tag{1}$$

where the *dimensionless* parameter η is actually a small number. If the ath resonator is located at position $\boldsymbol{x}_a$ ($|\boldsymbol{x}_a| = R$ for all $a = 1, \ldots, N$), where the outward unit normal is $\boldsymbol{n}_a$ and each one is modelled as a simple, non-damped harmonic oscillator then [1, 3]

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} - \mu \nabla^2 \boldsymbol{u} - (\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}) = \boldsymbol{f}(\boldsymbol{x}, t)$$

$$+\eta M \Omega^2 \sum_{a=1}^{N} \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}_a)\, [\xi_a(t) - \boldsymbol{n}_a \cdot \boldsymbol{u}(\boldsymbol{x}_a, t)]\, \boldsymbol{n}_a \tag{2}$$

$$\ddot{\xi}_a(t) = -\Omega^2\, [\xi_a(t) - \boldsymbol{n}_a \cdot \boldsymbol{u}(\boldsymbol{x}_a, t)]\,. \tag{3}$$

Equations (2) and (3) above constitute a rather complicated system of coupled differential equations, the solution to which, even if only *formal*, it is not possible to write down. Thankfully, though, we are only interested in practice in the *N measurable* quantities

$$q_a(t) \equiv \xi_a(t) - \boldsymbol{n}_a \cdot \boldsymbol{u}(\boldsymbol{x}_a, t), \qquad a = 1, \ldots, N \tag{4}$$

rather than in the *complete* solution. Even so, things are not completely straightforward, so we concentrate on what kind of solution is feasible and what its meaning is.

First of all we ought to make a choice of resonator frequency. Since GWs can only possibly couple to the sphere's monopole or quadrupole modes we shall choose one of the following:

$$\Omega = \omega_n 0 \qquad \text{or} \qquad \Omega = \omega_n 2. \tag{5}$$

Attachment of resonators to the system actually causes coupling of the sphere's modes other than the monopole and quadrupole modes among themselves and to the resonator set. These couplings can be analysed perturbatively in the small parameter η, and it can be seen [3] that they are very weak, except in those modes to which the resonators' frequency Ω is tuned. The effect consists in the *splitting* of the corresponding sphere's frequency into *one* symmetric doublet if monopole tuning ($\Omega = \omega_n 0$) is chosen or into *five* symmetric doublets if quadrupole tuning ($\Omega = \omega_n 2$) is chosen. The frequencies of these doublets are given by [3]

$$\omega_{a\pm}^2 = \omega_{nl}^2 \left(1 \pm \sqrt{\frac{2l+1}{4\pi}} A_{nl}(R) \zeta_a \eta^{1/2}\right) \tag{6}$$

where $l = 0$ or 2. Here, ζ_a^2 are the eigenvalues of the $N \times N$ symmetric matrix $P_l(\boldsymbol{n}_a \cdot \boldsymbol{n}_b)$. It can be shown that there are $2l + 1$ non-null such eigenvalues when $N \geqslant 2l + 1$. This results in *one* single doublet if $l = 0$ and in *five* doublets if $l = 2$ as announced above. This is so for *all* resonator configurations; if the one actually chosen has *symmetries*, some of the members of different doublets may fall on top of one another, i.e. *degeneracy* arises.

This is what happens, for example, with the TIGA configuration proposed in [4], where the five doublets collapse into a single one. The generality and simplicity of our analysis has enabled us to search for alternative layouts which may offer advantages with respect to the quoted TIGA. The most interesting one we have found is displayed schematically in figure 1. It is based on a *polyhedric* shape, following the philosophy of having flat faces for ease of mounting and manipulation. Our polyhedron, however, has 60 identical

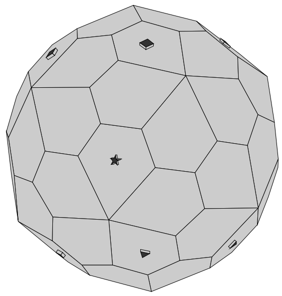

Figure 1. The proposed polyhedric antenna. Transducers are marked as follows: *squares*, on the *upper* region of the display, for the first quadrupole frequency, *triangles*, on the *lower* part, for the second quadrupole frequency and a *star* for the monopole.

faces, which makes it considerably more spherical than the 32-face TI, and which therefore permits the attachment of more resonators than just five or six, a very attractive possibility as we now discuss.

In our proposal there are a total of 11 transducers: five of them tuned to the *first quadrupole* frequency of the sphere, another five to the *second* one and an 11th resonator tuned to the lowest *monopole* frequency of the sphere. Such a configuration is meant to take advantage of the high *cross section* of a spherical antenna for absorption of GW energy at its first *two* quadrupole modes (see [5] for a more complete discussion of this point), and is also intended to enable measurement, or thresholding, of eventual *monopole GW radiation*—not predicted by general relativity, but by other theories of the gravitational interaction.

The two sets of five resonators are placed on five faces every 72° around one of the axes of pentagonal symmetry of our polyhedron, whilst the 11th can be placed anywhere—see figure 1. This has a good advantage relative to the TIGA in that fewer transducers per mode sensed are needed (five rather than six), but also in the structure of the system's *mode channels*: these are in this case such that *each of the five quadrupole GW modes couples to one of the frequency doublets of the coupled antenna system*. This can be an extremely appealing feature of a GW detector, as different *wave amplitudes* are seen at different *detector frequencies*.

The results of the above sketched analysis are not only remarkably elegant, they have also been checked against the experimental data produced by the prototype experiment carried out at Louisiana State University [4]; agreement between theory and measurement is observed to *four decimal places*. We shall not, however, give further details here, as the actual analysis needs to include a suitable procedure to address the breaking of spherical symmetry caused by the system's *suspension device* in the laboratory and this requires somewhat more sophisticated theoretical support.

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References

[1] Landau L D and Lifshitz E M 1970 *Theory of Elasticity* (Oxford: Pergamon)
[2] Lobo J A 1995 *Phys. Rev.* D **52** 591
[3] Lobo J A and Serrano M A 1996 *Europhys. Lett.* **35** 253
[4] Johnson W and Merkowitz S M 1993 *Phys. Rev. Lett.* **70** 2367
Johnson W and Merkowitz S M 1995 *Phys. Rev.* D **51** 2546
[5] Coccia E, Lobo J A and Ortega J A 1995 *Phys. Rev.* D **52** 3735